Logiweb codex of logic

Up Help

logic, $\mathrm{MP}'_{\mathrm{d}}$, $\mathrm{A1}'_{\mathrm{i}}$, Hypothesize, $\mathrm{A2}'_{\mathrm{i}}$, $\mathrm{A2}'_{\mathrm{ii}}$, $\mathrm{MP}'_{\mathrm{h}}$, $\mathrm{A2}'_{\mathrm{iid}}$, $\mathrm{MP}'_{\mathrm{hd}}$, Mendelson **1.8**, Mendelson $\mathbf{1.8}_{\mathrm{i}}$, Repetition, Mendelson **1.47** b, Mendelson **1.47** c, Mendelson **1.47** e, Mendelson **1.11** d, $\mathrm{A1}'_{\mathrm{ih}}$, $\mathrm{A2}'_{\mathrm{ih}}$, $\mathrm{A2}'_{\mathrm{iih}}$, Mendelson $\mathbf{1.47}\ \mathrm{b}_{\mathrm{h}}$, Mendelson $\mathbf{1.47}\ \mathrm{c}_{\mathrm{h}}$, Mendelson **1.11** c, Mendelson **1.48** d, Mendelson **1.48** e, Mendelson **1.48** h, Mendelson **1.10** a, Mendelson **1.10** b, $\mathrm{S1}'_{\mathrm{i}}$, $\mathrm{S1}'_{\mathrm{ii}}$, $\mathrm{S2}'_{\mathrm{i}}$, $\mathrm{S2}'_{\mathrm{ih}}$, $\mathrm{S9}'_{\mathrm{ii}}$, Induction, Mendelson **3.2** a, Mendelson **3.2** b, Mendelson $\mathbf{3.2}\ \mathrm{b}_{\mathrm{i}}$, Mendelson **3.2** c, Mendelson $\mathbf{3.2}\ \mathrm{c}_{\mathrm{ii}}$, Mendelson $\mathbf{3.2}\ \mathrm{c}_{\mathrm{iih}}$, Mendelson **3.2** d, Mendelson $\mathbf{3.2}\ \mathrm{d}_{\mathrm{ii}}$, Mendelson $\mathbf{3.2}\ \mathrm{d}_{\mathrm{iih}}$, Mendelson **3.2** f, Mendelson **3.2** f i, Mendelson **3.2** f ii, Mendelson **3.2** g, Mendelson **3.2** g i, Mendelson **3.2** g ii, Mendelson **3.2** h, Mendelson **3.2** h i, Mendelson **3.2** h ii, hyp, instance, conclusion, $* \trianglerighteq *$, $* \trianglerighteq * \trianglerighteq *$, $* \trianglerighteq_{\mathrm{h}} *$, $* \trianglerighteq_{\mathrm{h}} * \trianglerighteq_{\mathrm{h}} *$, Line $* : \bullet \ * \trianglerighteq * \gg *; *$, Line $* : \bullet \ * \circ \trianglerighteq * \gg *; *$, Line $* : \bullet \ * \trianglerighteq * \circ \gg *; *$, Line $* : \bullet \ * \trianglerighteq * \trianglerighteq * \gg *; *$, Line $* : \bullet$ Hypothesis $\gg *; *$, Line $* : \bullet * \gg *; *$, Line $* : \bullet * \gg *; *$,

logic

[logic $\overset{\text{prio}}{\longrightarrow}$
Preassociative
[logic], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** $[*]$], [x], [y], [z], $[[* \bowtie *]]$, $[[* \overset{*}{\longrightarrow} *]]$, [pyk], [tex], [name], [prio], $[*]$, $[\mathsf{T}]$, $[\mathrm{if}(*, *, *)]$, $[[* \overset{*}{\Rightarrow} *]]$, [val], [claim], $[\bot]$, $[\mathrm{f}(*)]$, $[(*)^{\mathrm{I}}]$, $[\mathsf{F}]$, [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\mathrm{M}}]$, $[\mathrm{If}(*, *, *)]$, [array{$*$} $*$ end array], [l], [c], [r], [empty], $[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$, $[\mathcal{U}^{\mathrm{M}}(*)]$, $[\mathbf{apply}(*, *)]$, $[\mathbf{apply}_1(*, *)]$, [identifier$(*)$], [identifier$_1(*, *)$], [array-plus$(*, *)$], [array-remove$(*, *, *)$], [array-put$(*, *, *, *)$], [array-add$(*, *, *, *, *)$], [bit$(*, *)$], [bit$_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], $[\mathcal{E}(*, *, *)]$, $[\mathcal{E}_2(*, *, *, *, *)]$, $[\mathcal{E}_3(*, *, *, *)]$, $[\mathcal{E}_4(*, *, *, *)]$, $[\mathbf{lookup}(*, *, *)]$, $[\mathbf{abstract}(*, *, *, *)]$, $[\lceil * \rceil]$, $[\mathcal{M}(*, *, *)]$, $[\mathcal{M}_2(*, *, *, *)]$, $[\mathcal{M}^*(*, *, *)]$, [macro], $[\mathrm{s}_0]$, $[\mathbf{zip}(*, *)]$, $[\mathbf{assoc}_1(*, *, *)]$, $[(*)^{\mathbf{P}}]$, [self], $[[* \doteq *]]$, $[[* \doteq *]]$, $[[* \doteq *]]$, $[[* \overset{\text{pyk}}{=} *]]$, $[[* \overset{\text{tex}}{=} *]]$, $[[* \overset{\text{name}}{=} *]]$, [**Priority table**$[*]$], $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$, $[\tilde{\mathcal{M}}_4(*, *, *, *)]$, $[\tilde{\mathcal{M}}(*, *, *)]$, $[\tilde{\mathcal{Q}}(*, *, *)]$, $[\tilde{\mathcal{Q}}_2(*, *, *)]$, $[\tilde{\mathcal{Q}}_3(*, *, *, *)]$, $[\tilde{\mathcal{Q}}^*(*, *, *)]$, $[(*)]$, $[\mathbf{aspect}(*, *)]$, $[\mathbf{aspect}(*, *, *)]$, $[\langle * \rangle]$, $[\mathbf{tuple}_1(*)]$, $[\mathbf{tuple}_2(*)]$, $[\mathrm{let}_2(*, *)]$,

[$\mathrm{let}_1(*,*)$], [[$* \overset{\text{claim}}{=} *$]], [checker], [**check**$(*,*)$], [**check**$_2(*,*,*)$], [**check**$_3(*,*,*)$], [**check**$^*(*,*)$], [**check**$^*_2(*,*,*)$], [[$*$]$^{\cdot}$], [[$*$]$^{-}$], [[$*$]$^{\circ}$], [msg], [[$* \overset{\text{msg}}{=} *$]], [<stmt>], [stmt], [[$* \overset{\text{stmt}}{=} *$]], [HeadNil$'$], [HeadPair$'$], [Transitivity$'$], [$\perp\!\!\!\perp$], [Contra$'$], [$\mathrm{T}'_{\mathrm{E}}$], [$\mathrm{L}_1$], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle {}^{*} * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathrm{v}}$], [intro$(*,*,*,*)$], [intro$(*,*,*)$], [error$(*,*)$], [error$_2(*,*)$], [proof$(*,*,*)$], [proof$_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\mathrm{I}}(*,*)$], [$\mathcal{S}^{\triangleright}(*,*)$], [$\mathcal{S}^{\triangleright}_1(*,*,*)$], [$\mathcal{S}^{\mathrm{E}}(*,*)$], [$\mathcal{S}^{\mathrm{E}}_1(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}^{+}_1(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}^{-}_1(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}^{*}_1(*,*,*)$], [$\mathcal{S}^{*}_2(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}^{@}_1(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}^{\vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}^{\Vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}\cdot}(*,*)$], [$\mathcal{S}^{\mathrm{i.e.}\cdot}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}\cdot}_2(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}^{\forall}_1(*,*,*,*)$], [$\mathcal{S}^{;}(*,*)$], [$\mathcal{S}^{;}_1(*,*,*)$], [$\mathcal{S}^{;}_2(*,*,*,*)$], [$\mathcal{T}(*)$], [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut$(*,*)$], [Head$_{\oplus}(*)$], [Tail$_{\oplus}(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [T_{E}], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(* = *,*)$], [unify$^*(* = *,*)$], [unify$_2(* = *,*)$], [L_{a}], [L_{b}], [L_{c}], [L_{d}], [L_{e}], [L_{f}], [L_{g}], [L_{h}], [L_{i}], [L_{j}], [L_{k}], [L_{l}], [L_{m}], [L_{n}], [L_{o}], [L_{p}], [L_{q}], [L_{r}], [L_{s}], [L_{t}], [L_{u}], [L_{v}], [L_{w}], [L_{x}], [L_{y}], [L_{z}], [L_{A}], [L_{B}], [L_{C}], [L_{D}], [L_{E}], [L_{F}], [L_{G}], [L_{H}], [L_{I}], [L_{J}], [L_{K}], [L_{L}], [L_{M}], [L_{N}], [L_{O}], [L_{P}], [L_{Q}], [L_{R}], [L_{S}], [L_{T}], [L_{U}], [L_{V}], [L_{W}], [L_{X}], [L_{Y}], [L_{Z}], [$\mathrm{L}_{?}$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$\mathrm{p}_0$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], [peano], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [nonfree$(*,*)$], [nonfree$^*(*,*)$], [free$\langle * | * := * \rangle$], [free$^*\langle * | * := * \rangle$], [$* \equiv \langle * | * := * \rangle$], [$* \equiv \langle {}^{*} * | * := * \rangle$], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$], [$\mathrm{MP}'_{\mathrm{d}}$], [$\mathrm{A1}'_{\mathrm{i}}$], [Hypothesize], [$\mathrm{A2}'_{\mathrm{i}}$], [$\mathrm{A2}'_{\mathrm{ii}}$], [$\mathrm{MP}'_{\mathrm{h}}$], [$\mathrm{A2}'_{\mathrm{iid}}$], [$\mathrm{MP}'_{\mathrm{hd}}$], [Mendelson **1.8**], [Mendelson **1.8**$_{\mathrm{i}}$], [Repetition], [Mendelson **1.47** b], [Mendelson **1.47** c], [Mendelson **1.47** e], [Mendelson **1.11** d], [$\mathrm{A1}'_{\mathrm{ih}}$], [$\mathrm{A2}'_{\mathrm{ih}}$], [$\mathrm{A2}'_{\mathrm{iih}}$], [Mendelson **1.47** b$_{\mathrm{h}}$], [Mendelson **1.47** c$_{\mathrm{h}}$], [Mendelson **1.11** c], [Mendelson **1.48** d], [Mendelson **1.48** e], [Mendelson **1.48** h], [Mendelson **1.10** a], [Mendelson **1.10** b], [$\mathrm{S1}'_{\mathrm{i}}$], [$\mathrm{S1}'_{\mathrm{ii}}$], [$\mathrm{S2}'_{\mathrm{i}}$], [$\mathrm{S2}'_{\mathrm{ih}}$], [$\mathrm{S9}'_{\mathrm{ii}}$], [Induction], [Mendelson **3.2** a], [Mendelson **3.2** b], [Mendelson **3.2** b$_{\mathrm{i}}$], [Mendelson **3.2** c], [Mendelson **3.2** c$_{\mathrm{ii}}$], [Mendelson **3.2** c$_{\mathrm{iih}}$], [Mendelson **3.2** d], [Mendelson **3.2** d$_{\mathrm{ii}}$], [Mendelson **3.2** d$_{\mathrm{iih}}$], [Mendelson **3.2** f], [Mendelson **3.2** f i], [Mendelson **3.2** f ii], [Mendelson **3.2** g], [Mendelson **3.2** g i], [Mendelson **3.2** g ii], [Mendelson **3.2** h], [Mendelson **3.2** h i], [Mendelson **3.2** h ii], [hyp], [instance], [conclusion];

Preassociative

[$*$_\{$*$\}], [$*'$], [$*[\,*\,]$], [$*[*\rightarrow *]$], [$*[*\Rightarrow *]$], [$\dot{*}$];

Preassociative
[“ ∗ ”], [], [$(*)^{\mathbf{t}}$], [string(∗) + ∗], [string(∗) ++ ∗], [
∗], [∗], [!∗], [”∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗],
[0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗],
[@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗],
[O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [ˆ ∗],
[_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗],
[p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [˜ ∗],
[**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end],
[newline ∗], [macro newline ∗];
Preassociative
[∗0], [∗1], [0b], [∗-color(∗)], [∗-color*(∗)];
Preassociative
[∗ ’ ∗], [∗ ‘ ∗];
Preassociative
[$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^{0}$],
[$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*'$];
Preassociative
[∗ · ∗], [∗ $\cdot_0$ ∗], [∗ $\dot{:}$ ∗];
Preassociative
[∗ + ∗], [∗ $+_0$ ∗], [∗ $+_1$ ∗], [∗ − ∗], [∗ $-_0$ ∗], [∗ $-_1$ ∗], [∗ $\dot{+}$ ∗];
Preassociative
[∗ ∪ {∗}], [∗ ∪ ∗], [∗\{∗}];
Postassociative
[∗ ∴ ∗], [∗ $\dot{\therefore}$ ∗], [∗ $\underline{::}$ ∗], [∗ $\underline{+2*}$ ∗], [∗ : : ∗], [∗ +2∗ ∗];
Postassociative
[∗, ∗];
Preassociative
[∗ $\overset{\mathrm{B}}{\approx}$ ∗], [∗ $\overset{\mathrm{D}}{\approx}$ ∗], [∗ $\overset{\mathrm{C}}{\approx}$ ∗], [∗ $\overset{\mathrm{P}}{\approx}$ ∗], [∗ ≈ ∗], [∗ = ∗], [∗ $\overset{+}{\to}$ ∗], [∗ $\overset{\mathrm{t}}{=}$ ∗], [∗ $\overset{\mathrm{t}^*}{=}$ ∗], [∗ $\overset{\mathrm{r}}{=}$ ∗],
[∗ $\in_{\mathrm{t}}$ ∗], [∗ $\subseteq_{\mathrm{T}}$ ∗], [∗ $\overset{\mathrm{T}}{=}$ ∗], [∗ $\overset{\mathrm{s}}{=}$ ∗], [∗ free in ∗], [∗ free in* ∗], [∗ free for ∗ in ∗],
[∗ free for* ∗ in ∗], [∗ $\in_{\mathrm{c}}$ ∗], [∗ < ∗], [∗ $<'$ ∗], [∗ $\leq'$ ∗], [∗ $\overset{\mathrm{p}}{=}$ ∗], [$*^{\mathcal{P}}$];
Preassociative
[¬∗], [$\dot{\neg}$ ∗];
Preassociative
[∗ ∧ ∗], [∗ $\ddot{\wedge}$ ∗], [∗ $\tilde{\wedge}$ ∗], [∗ $\wedge_{\mathrm{c}}$ ∗], [∗ $\dot{\wedge}$ ∗];
Preassociative
[∗ ∨ ∗], [∗ ∥ ∗], [∗ $\ddot{\vee}$ ∗], [∗ $\dot{\vee}$ ∗];
Preassociative
[$\dot{\forall}$∗: ∗], [$\dot{\exists}$∗: ∗];
Postassociative
[∗ $\ddot{\Rightarrow}$ ∗], [∗ $\dot{\Rightarrow}$ ∗], [∗ $\dot{\Leftrightarrow}$ ∗];
Postassociative
[∗ : ∗], [∗!∗];
Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$
Preassociative
$[\lambda * .*], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *];$
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}];$
Preassociative
$[* @ *], [* \rhd *], [* \blacktriangleright *], [* \gg *], [* \trianglerighteq *], [* \trianglerighteq * \trianglerighteq *], [* \trianglerighteq_{\mathrm{h}} *], [* \trianglerighteq_{\mathrm{h}} * \trianglerighteq_{\mathrm{h}} *];$
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
$[\forall *: *];$
Postassociative
$[* \oplus *];$
Postassociative
$[*; *];$
Preassociative
$[* \text{ proves } *];$
Preassociative
$[* \textbf{ proof of } * : *], [\text{Line } * : * \gg *; *], [\text{Last line } * \gg * \Box],$
$[\text{Line } * : \text{Premise} \gg *; *], [\text{Line } * : \text{Side-condition} \gg *; *], [\text{Arbitrary} \gg *; *],$
$[\text{Local} \gg * = *; *], [\text{Line } * : \bullet \ * \trianglerighteq * \gg *; *], [\text{Line } * : \bullet \ * \circ \trianglerighteq * \gg *; *],$
$[\text{Line } * : \bullet \ * \trianglerighteq * \circ \gg *; *], [\text{Line } * : \bullet \ * \trianglerighteq * \trianglerighteq * \gg *; *],$
$[\text{Line } * : \bullet \text{ Hypothesis} \gg *; *], [\text{Line } * : \bullet * \gg *; *], [\text{Line } * : \bullet * \gg *; *];$
Postassociative
$[* \text{ then } *], [*[*]*];$
Preassociative
$[*\&*];$
Preassociative
$[*\backslash\backslash *];]$

[logic $\overset{\text{pyk}}{\to}$ "logic"]

$\mathrm{MP}'_{\mathrm{d}}$

$[\mathrm{MP}'_{\mathrm{d}} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}: \forall \underline{\mathsf{b}}: \forall \underline{\mathsf{c}}: [\ [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \vdash\ [\ \underline{\mathsf{a}} \vdash\ [\ \underline{\mathsf{b}} \vdash\ [\ [\ [\ [\ \mathrm{MP}' \rhd\ [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \rhd \underline{\mathsf{a}}\ ]\ \gg\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \rhd \underline{\mathsf{b}}\ ]\ \gg \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MP}'_{\mathrm{d}} \overset{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}: \forall \underline{\mathsf{b}}: \forall \underline{\mathsf{c}}: [\ [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \vdash\ [\ \underline{\mathsf{a}} \vdash\ [\ \underline{\mathsf{b}} \vdash \underline{\mathsf{c}}\ ]\ ]\ ]$

$[\mathrm{MP}'_{\mathrm{d}} \overset{\text{tex}}{\to} \text{"MP'\_\{d\}"}]$

$[\mathrm{MP}'_{\mathrm{d}} \overset{\text{pyk}}{\to} \text{"double rule prime mp"}]$

$\mathrm{A1}'_{\mathrm{i}}$

$[\mathrm{A1}'_{\mathrm{i}} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\, \underline{\mathsf{a}} \vdash [\, [\, \mathrm{A1}' \gg [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \,] \,] \,] \,] \,] \,;\, [\, [\, [\, \mathrm{MP}' \rhd [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \,] \,] \,] \rhd \underline{\mathsf{a}} \,] \gg [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \,] \,] \,] \,] \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A1}'_{\mathrm{i}} \overset{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\, \underline{\mathsf{a}} \vdash [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \,] \,] \,]$

$[\mathrm{A1}'_{\mathrm{i}} \overset{\text{tex}}{\to} \text{“A1'\_\{i\}”}]$

$[\mathrm{A1}'_{\mathrm{i}} \overset{\text{pyk}}{\to} \text{“inference axiom prime a one”}]$

Hypothesize

$[\mathrm{Hypothesize} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{Hypothesize} \doteqdot \mathrm{A1}'_{\mathrm{i}}] \rceil)]$

$[\mathrm{Hypothesize} \overset{\text{tex}}{\to} \text{“Hypothesize”}]$

$[\mathrm{Hypothesize} \overset{\text{pyk}}{\to} \text{“rule hypothesize”}]$

$\mathrm{A2}'_{\mathrm{i}}$

$[\mathrm{A2}'_{\mathrm{i}} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\, [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \vdash [\, [\, \mathrm{A2}' \gg [\, [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \Rightarrow [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \Rightarrow [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \,] \,;\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \Rightarrow [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \Rightarrow [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \rhd [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \gg [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \Rightarrow [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \,] \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A2}'_{\mathrm{i}} \overset{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\, [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \vdash [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \Rightarrow [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \,]$

$[\mathrm{A2}'_{\mathrm{i}} \overset{\text{tex}}{\to} \text{“A2'\_\{i\}”}]$

$[\mathrm{A2}'_{\mathrm{i}} \overset{\text{pyk}}{\to} \text{“inference axiom prime a two”}]$

$\mathrm{A2}'_{\mathrm{ii}}$

$[\mathrm{A2}'_{\mathrm{ii}} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\, [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \vdash [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \vdash [\, [\, [\, \mathrm{A2}'_{\mathrm{i}} \rhd [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \gg [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \Rightarrow [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \,;\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \Rightarrow [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \rhd [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \,] \gg [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \,] \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A2}'_{\mathrm{ii}} \overset{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\, [\, \underline{\mathsf{a}} \Rightarrow [\, \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \,] \,] \vdash [\, [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \,] \vdash [\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \,] \,] \,] \,]$

$[\mathrm{A2}'_{\mathrm{ii}} \overset{\text{tex}}{\to} \text{“A2'\_\{ii\}”}]$

$[\mathrm{A2}'_{\mathrm{ii}} \overset{\text{pyk}}{\to} \text{“inference inference axiom prime a two”}]$

$\mathrm{MP}'_{\mathrm{h}}$

$[\mathrm{MP}'_{\mathrm{h}} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{MP}'_{\mathrm{h}} \doteq \mathrm{A2}'_{\mathrm{ii}}]\rceil)]$

$[\mathrm{MP}'_{\mathrm{h}} \overset{\text{tex}}{\to} \text{“MP'\_\{h\}”}]$

$[\mathrm{MP}'_{\mathrm{h}} \overset{\text{pyk}}{\to} \text{“hypothetical rule prime mp”}]$

$\mathrm{A2}'_{\mathrm{iid}}$

$[\mathrm{A2}'_{\mathrm{iid}} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon [\ [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{b}} \Rightarrow\ [\ \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ \vdash\ [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ \vdash\ [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ \vdash\ [\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ii}} \rhd [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{b}} \Rightarrow\ [\ \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ \gg [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ii}} \rhd [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \gg [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ ]\ ]\ \rfloor, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A2}'_{\mathrm{iid}} \overset{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon [\ [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{b}} \Rightarrow\ [\ \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ \vdash\ [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ \vdash\ [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ \vdash\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{d}}\ ]\ ]\ ]\ ]\ ]$

$[\mathrm{A2}'_{\mathrm{iid}} \overset{\text{tex}}{\to} \text{“A2'\_\{iid\}”}]$

$[\mathrm{A2}'_{\mathrm{iid}} \overset{\text{pyk}}{\to} \text{“double inference inference axiom prime a two”}]$

$\mathrm{MP}'_{\mathrm{hd}}$

$[\mathrm{MP}'_{\mathrm{hd}} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{MP}'_{\mathrm{hd}} \doteq \mathrm{A2}'_{\mathrm{iid}}]\rceil)]$

$[\mathrm{MP}'_{\mathrm{hd}} \overset{\text{tex}}{\to} \text{“MP'\_\{hd\}”}]$

$[\mathrm{MP}'_{\mathrm{hd}} \overset{\text{pyk}}{\to} \text{“double hypothetical rule prime mp”}]$

Mendelson **1.8**

$[\text{Mendelson } \mathbf{1.8} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{a}} \Rightarrow\ [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\ ]\ \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ii}} \rhd [\ \underline{\mathsf{a}} \Rightarrow\ [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\ ]\ \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \Rightarrow\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ \rfloor, \mathrm{p}_0, \mathsf{c})]$

$[\text{Mendelson } \mathbf{1.8} \overset{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]$

$[\text{Mendelson } \mathbf{1.8} \overset{\text{tex}}{\to} \text{“Mendelson\textbackslash\ \textbackslash textbf\{1.8\}”}]$

$[\text{Mendelson } \mathbf{1.8} \overset{\text{pyk}}{\to} \text{“mendelson lemma one eight”}]$

Mendelson $\mathbf{1.8_i}$

$[\text{Mendelson } \mathbf{1.8_i} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon [\ \underline{a} \vdash [\ [\ \text{Mendelson } \mathbf{1.8} \gg [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ ;\ [\ [\ [\ \text{MP}' \rhd [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ \rhd \underline{a}\ ]\ \gg \underline{a}\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\text{Mendelson } \mathbf{1.8_i} \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon [\ \underline{a} \vdash \underline{a}\ ]\ ]$

$[\text{Mendelson } \mathbf{1.8_i} \overset{\text{tex}}{\rightarrow} \text{“Mendelson\ \textbf\{1.8\}\_\{i\}”}]$

$[\text{Mendelson } \mathbf{1.8_i} \overset{\text{pyk}}{\rightarrow} \text{“inference mendelson lemma one eight”}]$

Repetition

$[\text{Repetition} \overset{\text{macro}}{\rightarrow} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\text{Repetition} \doteq \text{Mendelson } \mathbf{1.8_i}]\rceil)]$

$[\text{Repetition} \overset{\text{tex}}{\rightarrow} \text{“Repetition”}]$

$[\text{Repetition} \overset{\text{pyk}}{\rightarrow} \text{“rule repetition”}]$

Mendelson **1.47** b

$[\text{Mendelson } \mathbf{1.47} \text{ b} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \vdash [\ [\ \underline{b} \Rightarrow \underline{c}\ ] \vdash [\ [\ [\ \text{A1}_i' \rhd [\ \underline{b} \Rightarrow \underline{c}\ ]\ ] \gg [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{A2}'_{ii} \rhd [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ \rhd [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ \gg [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\text{Mendelson } \mathbf{1.47} \text{ b} \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \vdash [\ [\ \underline{b} \Rightarrow \underline{c}\ ] \vdash [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]$

$[\text{Mendelson } \mathbf{1.47} \text{ b} \overset{\text{tex}}{\rightarrow} \text{“Mendelson\ \textbf\{1.47\}\ b”}]$

$[\text{Mendelson } \mathbf{1.47} \text{ b} \overset{\text{pyk}}{\rightarrow} \text{“mendelson exercise one fourtyseven b”}]$

Mendelson **1.47** c

$[\text{Mendelson } \mathbf{1.47} \text{ c} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \vdash [\ [\ [\ \text{A2}_i' \rhd [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ \gg [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{A1}_i' \rhd [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ \gg [\ \underline{b} \Rightarrow [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{A1}' \gg [\ \underline{b} \Rightarrow [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{A2}'_{ii} \rhd [\ \underline{b} \Rightarrow [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{b} \Rightarrow [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ]\ \gg [\ \underline{b} \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\text{Mendelson } \mathbf{1.47} \text{ c} \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \vdash [\ \underline{b} \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]$

$[\text{Mendelson } \mathbf{1.47} \text{ c} \overset{\text{tex}}{\rightarrow} \text{“Mendelson\ \textbf\{1.47\}\ c”}]$

$[\text{Mendelson } \mathbf{1.47} \text{ c} \overset{\text{pyk}}{\rightarrow} \text{“mendelson exercise one fourtyseven c”}]$

Mendelson **1.47** e

[Mendelson **1.47** e $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ] \vdash [\ \underline{b} \vdash [\ [\ [$ Mendelson **1.47** c $\triangleright$ $[\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ] \gg [\ \underline{b} \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ;\ [\ [\ [$ MP$'$ $\triangleright$ $[\ \underline{b} \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ \triangleright \underline{b}\ ] \gg [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ ], p$_0$, c)]

[Mendelson **1.47** e $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ] \vdash [\ \underline{b} \vdash [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]$

[Mendelson **1.47** e $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{1.47}\ e"]

[Mendelson **1.47** e $\overset{\text{pyk}}{\longrightarrow}$ "mendelson exercise one fourtyseven e"]

Mendelson **1.11** d

[Mendelson **1.11** d $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [$ Mendelson **1.8** $\gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ]\ ]\ ]\ ;\ [\ [$ A3$' \gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ;\ [\ [\ [$ A1$'_{\text{i}} \triangleright [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ] \gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ;\ [\ [$ A1$' \gg [\ \underline{b} \Rightarrow [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ]\ ]\ ]\ ;\ [\ [\ [$ A1$'_{\text{i}} \triangleright [\ \underline{b} \Rightarrow [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ]\ ]\ ]\ ] \gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ \underline{b} \Rightarrow [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$ A2$'_{\text{ii}} \triangleright [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ] \triangleright [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ]\ ]\ ] \gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ;\ [\ [\ [\ [$ Mendelson **1.47** b$_{\text{h}} \triangleright [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ \underline{b} \Rightarrow [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ]\ ]\ ]\ ]\ ] \triangleright [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ] \gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ;\ [\ [$ Mendelson **1.8**$_{\text{i}} \triangleright [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ] \gg [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ], p$_0$, c)]

[Mendelson **1.11** d $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \dot{\neg}\underline{a}\ ] \Rightarrow \dot{\neg}\underline{b}\ ] \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]$

[Mendelson **1.11** d $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{1.11}\ d"]

[Mendelson **1.11** d $\overset{\text{pyk}}{\longrightarrow}$ "mendelson lemma one eleven d"]

A1$'_{\text{ih}}$

[A1$'_{\text{ih}}$ $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{h} \Rightarrow \underline{a}\ ] \vdash [\ [$ Mendelson **1.8** $\gg [\ \underline{h} \Rightarrow \underline{h}\ ]\ ]\ ;\ [\ [$ A1$' \gg [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ;\ [\ [\ [$ A1$'_{\text{i}} \triangleright [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$ A2$'_{\text{ii}} \triangleright [\ \underline{h} \Rightarrow [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ] \triangleright [\ \underline{h} \Rightarrow \underline{a}\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ;\ [\ [$ Mendelson **1.8**$_{\text{i}} \triangleright [\ \underline{h} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ], p$_0$, c)]

[A1$'_{\text{ih}}$ $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{h} \Rightarrow \underline{a}\ ] \vdash [\ \underline{h} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ]$

[A1$'_{\text{ih}}$ $\overset{\text{tex}}{\longrightarrow}$ "A1'_{ih}"]

[$\mathrm{A1}'_{\mathrm{ih}} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical inference axiom prime a one”]

$\mathrm{A2}'_{\mathrm{ih}}$

[$\mathrm{A2}'_{\mathrm{ih}} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \vdash [\ [$ Mendelson **1.8** $\gg [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{h}}\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \rhd [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ii}} \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [$ Mendelson **1.8**$_{\mathrm{i}} \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\rfloor, \mathrm{p}_0, \mathsf{c})$]

[$\mathrm{A2}'_{\mathrm{ih}} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \vdash [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$]

[$\mathrm{A2}'_{\mathrm{ih}} \overset{\mathrm{tex}}{\rightarrow}$ “A2'_{ih}”]

[$\mathrm{A2}'_{\mathrm{ih}} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical inference axiom prime a two”]

$\mathrm{A2}'_{\mathrm{iih}}$

[$\mathrm{A2}'_{\mathrm{iih}} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \vdash [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ \vdash [\ [$ Mendelson **1.8** $\gg [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{h}}\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \rhd [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ [\ \mathrm{A2}'_{\mathrm{iid}} \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \Rightarrow [\ [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ;\ [\ [$ Mendelson **1.8**$_{\mathrm{i}} \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\rfloor, \mathrm{p}_0, \mathsf{c})$]

[$\mathrm{A2}'_{\mathrm{iih}} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \vdash [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ \vdash [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$]

[$\mathrm{A2}'_{\mathrm{iih}} \overset{\mathrm{tex}}{\rightarrow}$ “A2'_{iih}”]

[$\mathrm{A2}'_{\mathrm{iih}} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical inference inference axiom prime a two”]

Mendelson **1.47** b_{h}

[Mendelson **1.47** $\mathrm{b}_{\mathrm{h}} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ \vdash [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ \vdash [\ [$ Mendelson **1.8** $\gg [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{h}}\ ]\ ]\ ;\ [\ [\ [\ \mathrm{A1}'_{\mathrm{ih}} \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{A2}'_{\mathrm{iih}} \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow [$

$\underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ \triangleright\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ];\ [\ [$ Mendelson $\mathbf{1.8}_{\mathrm{i}} \triangleright\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ], \mathrm{p}_0, \mathsf{c})]$

[Mendelson **1.47** $\mathrm{b_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon\ [\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ ] \vdash\ [\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ] \vdash\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]$

[Mendelson **1.47** $\mathrm{b_h} \overset{\mathrm{tex}}{\rightarrow}$ "Mendelson\ \textbf{1.47}\ b_{h}"]

[Mendelson **1.47** $\mathrm{b_h} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical mendelson exercise one fourtyseven b"]

Mendelson **1.47** $\mathrm{c_h}$

[Mendelson **1.47** $\mathrm{c_h} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon\ [\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ] \vdash\ [\ [$ Mendelson $\mathbf{1.8} \gg\ [\ \underline{h} \Rightarrow \underline{h}\ ]\ ];\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ih}} \triangleright\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{ih}} \triangleright\ [\ \underline{h} \Rightarrow\ [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ];\ [\ [$ $\mathrm{A1}' \gg\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \triangleright\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ]\ ];\ [\ [\ [\ [\ \mathrm{A2}'_{\mathrm{iih}} \triangleright\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ] \triangleright\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ];\ [\ [$ Mendelson $\mathbf{1.8}_{\mathrm{i}} \triangleright\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ] \gg\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ], \mathrm{p}_0, \mathsf{c})]$

[Mendelson **1.47** $\mathrm{c_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon\ [\ [\ \underline{h} \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ] \vdash\ [\ \underline{h} \Rightarrow\ [\ \underline{b} \Rightarrow\ [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]$

[Mendelson **1.47** $\mathrm{c_h} \overset{\mathrm{tex}}{\rightarrow}$ "Mendelson\ \textbf{1.47}\ c_{h}"]

[Mendelson **1.47** $\mathrm{c_h} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical mendelson exercise one fourtyseven c"]

Mendelson **1.11** c

[Mendelson **1.11** c $\overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon\ [\ [$ Mendelson $\mathbf{1.8} \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow \neg \underline{a}\ ]\ ];\ [\ [$ Mendelson $\mathbf{1.8} \gg\ [\ \underline{a} \Rightarrow \underline{a}\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \triangleright\ [\ \underline{a} \Rightarrow \underline{a}\ ]\ ] \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ ];\ [\ [\ \mathrm{A1}' \gg\ [\ \underline{a} \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \triangleright\ [\ \underline{a} \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ] \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ];\ [\ [\ \mathrm{A1}' \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ] \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{ih}} \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ]\ ]\ ] \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A2}'_{\mathrm{iih}} \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ ] \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ ] \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ \underline{a} \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \underline{a}\ ]\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ii}} \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ]\ ] \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow \neg \underline{a}\ ]\ ] \gg\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ];\ [\ [\ [\ \mathrm{A1}'_{\mathrm{ih}} \triangleright\ [\ [\ \neg \underline{a}\ ] \Rightarrow\ [\ [\ \neg \underline{b}\ ] \Rightarrow \neg \underline{a}\ ]\ ]\ ] \gg\ [\ [\ \neg \underline{a}$

] ⇒ [a ⇒ [[∴ b] ⇒ ∴ a]]] ; [[A3′ ≫ [[[∴ b] ⇒ ∴ a] ⇒ [[[∴ b] ⇒ a] ⇒ b]]] ; [[[A1′$_{\mathrm{i}}$ ▷ [[[∴ b] ⇒ ∴ a] ⇒ [[[∴ b] ⇒ a] ⇒ b]]] ≫ [[∴ a] ⇒ [[[∴ b] ⇒ ∴ a] ⇒ [[[∴ b] ⇒ a] ⇒ b]]] ; [[[A1′$_{\mathrm{ih}}$ ▷ [[∴ a] ⇒ [[[∴ b] ⇒ ∴ a] ⇒ [[[∴ b] ⇒ a] ⇒ b]]]] ≫ [[∴ a] ⇒ [a ⇒ [[[∴ b] ⇒ ∴ a] ⇒ [[[∴ b] ⇒ a] ⇒ b]]]] ; [[[[A2′$_{\mathrm{iih}}$ ▷ [[∴ a] ⇒ [a ⇒ [[[∴ b] ⇒ ∴ a] ⇒ [[[∴ b] ⇒ a] ⇒ b]]]]] ▷ [[∴ a] ⇒ [a ⇒ [[∴ b] ⇒ ∴ a]]]] ≫ [[∴ a] ⇒ [a ⇒ [[[∴ b] ⇒ a] ⇒ b]]]] ; [[[[A2′$_{\mathrm{iih}}$ ▷ [[∴ a] ⇒ [a ⇒ [[[∴ b] ⇒ a] ⇒ b]]]] ▷ [[∴ a] ⇒ [a ⇒ [[∴ b] ⇒ a]]]] ≫ [[∴ a] ⇒ [a ⇒ b]]] ; [[Mendelson **1.8**$_{\mathrm{i}}$ ▷ [[∴ a] ⇒ [a ⇒ b]]] ≫ [[∴ a] ⇒ [a ⇒ b], p$_0$, c)]

[Mendelson **1.11** c $\overset{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: [[∴ a] ⇒ [a ⇒ b]]]

[Mendelson **1.11** c $\overset{\text{tex}}{\rightarrow}$ “Mendelson\ \textbf{1.11}\ c”]

[Mendelson **1.11** c $\overset{\text{pyk}}{\rightarrow}$ “mendelson lemma one eleven c”]

Mendelson **1.48** d

[Mendelson **1.48** d $\overset{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: [[∴ [a ⇒ ∴ b]] ⇒ a]]

[Mendelson **1.48** d $\overset{\text{tex}}{\rightarrow}$ “Mendelson\ \textbf{1.48}\ d”]

[Mendelson **1.48** d $\overset{\text{pyk}}{\rightarrow}$ “mendelson exercise one fourtyeight d”]

Mendelson **1.48** e

[Mendelson **1.48** e $\overset{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: [[∴ [a ⇒ ∴ b]] ⇒ b]]

[Mendelson **1.48** e $\overset{\text{tex}}{\rightarrow}$ “Mendelson\ \textbf{1.48}\ e”]

[Mendelson **1.48** e $\overset{\text{pyk}}{\rightarrow}$ “mendelson exercise one fourtyeight e”]

Mendelson **1.48** h

[Mendelson **1.48** h $\overset{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: [a ⇒ [b ⇒ ∴ [a ⇒ ∴ b]]]]

[Mendelson **1.48** h $\overset{\text{tex}}{\rightarrow}$ “Mendelson\ \textbf{1.48}\ h”]

[Mendelson **1.48** h $\overset{\text{pyk}}{\rightarrow}$ “mendelson exercise one fourtyeight h”]

Mendelson **1.10** a

[Mendelson **1.10** a $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \vdash [\ [\ \underline{b} \Rightarrow \underline{c}\ ] \vdash [\ [\ \text{Mendelson } \mathbf{1.8} \gg [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ ;\ [\ [\ [\ [\ \text{Mendelson } \mathbf{1.47}\ b \rhd [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ \rhd [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ \gg [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ ;\ [\ [\ [\ [\ \text{Mendelson } \mathbf{1.47}\ b \rhd [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ \rhd [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \gg [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ;\ [\ [\ \text{Mendelson } \mathbf{1.8}_i \rhd [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ \gg [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[Mendelson **1.10** a $\overset{\text{stmt}}{\to}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow \underline{b}\ ] \vdash [\ [\ \underline{b} \Rightarrow \underline{c}\ ] \vdash [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]$]

[Mendelson **1.10** a $\overset{\text{tex}}{\to}$ "Mendelson\ \textbf{1.10}\ a"]

[Mendelson **1.10** a $\overset{\text{pyk}}{\to}$ "mendelson corollary one ten a"]

Mendelson **1.10** b

[Mendelson **1.10** b $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \vdash [\ \underline{b} \vdash [\ [\ \text{Mendelson } \mathbf{1.8} \gg [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ ;\ [\ [\ [\ [\ \text{Mendelson } \mathbf{1.47}\ b \rhd [\ \underline{a} \Rightarrow \underline{a}\ ]\ ]\ \rhd [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ \gg [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \text{Mendelson } \mathbf{1.47}\ e \rhd [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ \rhd \underline{b}\ ]\ \gg [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ;\ [\ [\ \text{Mendelson } \mathbf{1.8}_i \rhd [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ \gg [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[Mendelson **1.10** b $\overset{\text{stmt}}{\to}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow\ [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \vdash [\ \underline{b} \vdash [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]$]

[Mendelson **1.10** b $\overset{\text{tex}}{\to}$ "Mendelson\ \textbf{1.10}\ b"]

[Mendelson **1.10** b $\overset{\text{pyk}}{\to}$ "mendelson corollary one ten b"]

$S1'_i$

[$S1'_i \overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \vdash [\ [\ S1' \gg [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ MP' \rhd [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ]\ \gg [\ [\ \underline{a} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[$S1'_i \overset{\text{stmt}}{\to}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \vdash [\ [\ \underline{a} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ]$]

[$S1'_i \overset{\text{tex}}{\to}$ "S1'_{i}"]

[$S1'_i \overset{\text{pyk}}{\to}$ "inference axiom prime s one"]

$\mathrm{S1}'_{\mathrm{ii}}$

$[\mathrm{S1}'_{\mathrm{ii}} \overset{\mathrm{proof}}{\longrightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \vdash [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ] \vdash [\ [\ [\ \mathrm{S1}'_{\mathrm{i}} \rhd [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ ] \gg [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{b}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{b}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ] \gg [\ \underline{\mathsf{b}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{S1}'_{\mathrm{ii}} \overset{\mathrm{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \vdash [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ] \vdash [\ \underline{\mathsf{b}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$

$[\mathrm{S1}'_{\mathrm{ii}} \overset{\mathrm{tex}}{\longrightarrow} \text{“S1'\_\{ii\}”}]$

$[\mathrm{S1}'_{\mathrm{ii}} \overset{\mathrm{pyk}}{\longrightarrow} \text{“inference inference axiom prime s one”}]$

$\mathrm{S2}'_{\mathrm{i}}$

$[\mathrm{S2}'_{\mathrm{i}} \overset{\mathrm{proof}}{\longrightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \vdash [\ [\ \mathrm{S2}' \gg [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ ] \gg [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{S2}'_{\mathrm{i}} \overset{\mathrm{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \vdash [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]$

$[\mathrm{S2}'_{\mathrm{i}} \overset{\mathrm{tex}}{\longrightarrow} \text{“S2'\_\{i\}”}]$

$[\mathrm{S2}'_{\mathrm{i}} \overset{\mathrm{pyk}}{\longrightarrow} \text{“inference axiom prime s two”}]$

$\mathrm{S2}'_{\mathrm{ih}}$

$[\mathrm{S2}'_{\mathrm{ih}} \overset{\mathrm{proof}}{\longrightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon\ [\ [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ ] \vdash [\ [\ \text{Mendelson } \mathbf{1.8} \gg [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{h}}\ ]\ ]\ ;\ [\ [\ \mathrm{S2}' \gg [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{A1}'_{\mathrm{i}} \rhd [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{A2}'_{\mathrm{ii}} \rhd [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ;\ [\ [\ \text{Mendelson } \mathbf{1.8}_{\mathrm{i}} \rhd [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{S2}'_{\mathrm{ih}} \overset{\mathrm{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon\ [\ [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ ] \vdash [\ \underline{\mathsf{h}} \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]$

$[\mathrm{S2}'_{\mathrm{ih}} \overset{\mathrm{tex}}{\longrightarrow} \text{“S2'\_\{ih\}”}]$

$[\mathrm{S2}'_{\mathrm{ih}} \overset{\mathrm{pyk}}{\longrightarrow} \text{“hypothetical inference axiom prime s two”}]$

$\mathrm{S9}'_{\mathrm{ii}}$

$[\mathrm{S9}'_{\mathrm{ii}} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{y}\colon [\, \underline{b} \equiv \langle \underline{a} | \underline{y} := \dot{0} \rangle \Vdash [\, \underline{c} \equiv \langle \underline{a} | \underline{y} := \underline{y}' \rangle \Vdash [\, \underline{b} \vdash [\, [\, \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,]\,] \vdash [\, [\, [\, [\, \mathrm{S9}' \Rrightarrow \underline{b} \equiv \langle \underline{a} | \underline{y} := \dot{0} \rangle\,]\, \Rrightarrow \underline{c} \equiv \langle \underline{a} | \underline{y} := \underline{y}' \rangle\,]\, \gg [\, \underline{b} \dot{\Rightarrow} [\, [\, \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,]\, \dot{\Rightarrow} \dot{\forall} \underline{y}\colon \underline{a}\,]\,]\,] ; [\, [\, [\, [\, \mathrm{MP}'_{\mathrm{d}} \rhd [\, \underline{b} \dot{\Rightarrow} [\, [\, \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,]\, \dot{\Rightarrow} \dot{\forall} \underline{y}\colon \underline{a}\,]\,]\,] \rhd \underline{b}\,] \rhd \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,] \gg \dot{\forall} \underline{y}\colon \underline{a}\,]\,]\,]\,]\,]\,] \rceil, \mathrm{p}_0, \mathrm{c})]$

$[\mathrm{S9}'_{\mathrm{ii}} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{y}\colon [\, \underline{b} \equiv \langle \underline{a} | \underline{y} := \dot{0} \rangle \Vdash [\, \underline{c} \equiv \langle \underline{a} | \underline{y} := \underline{y}' \rangle \Vdash [\, \underline{b} \vdash [\, [\, \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,] \vdash \dot{\forall} \underline{y}\colon \underline{a}\,]\,]\,]\,]$

$[\mathrm{S9}'_{\mathrm{ii}} \overset{\mathrm{tex}}{\rightarrow} \text{“S9'\_\{ii\}”}]$

$[\mathrm{S9}'_{\mathrm{ii}} \overset{\mathrm{pyk}}{\rightarrow} \text{“inference inference axiom prime s nine”}]$

Induction

$[\mathrm{Induction} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{y}\colon [\, \underline{b} \equiv \langle \underline{a} | \underline{y} := \dot{0} \rangle \Vdash [\, \underline{c} \equiv \langle \underline{a} | \underline{y} := \underline{y}' \rangle \Vdash [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{a} \rceil | \lceil \underline{y} \rceil := \lceil \underline{y} \rceil \rangle \Vdash [\, \underline{b} \vdash [\, [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\, \vdash [\, [\, [\, \mathrm{Gen}' \rhd [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,]\, \gg \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,] ; [\, [\, [\, [\, [\, \mathrm{S9}'_{\mathrm{ii}} \Rrightarrow \underline{b} \equiv \langle \underline{a} | \underline{y} := \dot{0} \rangle\,]\, \Rrightarrow \underline{c} \equiv \langle \underline{a} | \underline{y} := \underline{y}' \rangle\,]\, \rhd \underline{b}\,] \rhd \dot{\forall} \underline{y}\colon [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\,]\, \gg \dot{\forall} \underline{y}\colon \underline{a}\,] ; [\, [\, [\, \mathrm{A4}' \Rrightarrow \lceil \underline{a} \rceil \equiv \langle \lceil \underline{a} \rceil | \lceil \underline{y} \rceil := \lceil \underline{y} \rceil \rangle\,]\, \gg [\, [\, \dot{\forall} \underline{y}\colon \underline{a}\,]\, \dot{\Rightarrow} \underline{a}\,]\,] ; [\, [\, [\, \mathrm{MP}' \rhd [\, [\, \dot{\forall} \underline{y}\colon \underline{a}\,]\, \dot{\Rightarrow} \underline{a}\,]\,] \rhd \dot{\forall} \underline{y}\colon \underline{a}\,]\, \gg \underline{a}\,]\,]\,]\,]\,]\,]\,]\,]\,]\,] \rceil, \mathrm{p}_0, \mathrm{c})]$

$[\mathrm{Induction} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{y}\colon [\, \underline{b} \equiv \langle \underline{a} | \underline{y} := \dot{0} \rangle \Vdash [\, \underline{c} \equiv \langle \underline{a} | \underline{y} := \underline{y}' \rangle \Vdash [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{a} \rceil | \lceil \underline{y} \rceil := \lceil \underline{y} \rceil \rangle \Vdash [\, \underline{b} \vdash [\, [\, \underline{a} \dot{\Rightarrow} \underline{c}\,]\, \vdash \underline{a}\,]\,]\,]\,]\,]$

$[\mathrm{Induction} \overset{\mathrm{tex}}{\rightarrow} \text{“Induction”}]$

$[\mathrm{Induction} \overset{\mathrm{pyk}}{\rightarrow} \text{“rule induction”}]$

Mendelson **3.2** a

$[\mathrm{Mendelson}\ \mathbf{3.2}\ \mathrm{a} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon [\, [\, \mathrm{S5}' \gg [\, [\, \underline{a} \dot{+} \dot{0}\,]\, \overset{\mathrm{p}}{=} \underline{a}\,]\,] ; [\, [\, [\, \mathrm{S1}'_{\mathrm{ii}} \rhd [\, [\, \underline{a} \dot{+} \dot{0}\,]\, \overset{\mathrm{p}}{=} \underline{a}\,]\,] \rhd [\, [\, \underline{a} \dot{+} \dot{0}\,]\, \overset{\mathrm{p}}{=} \underline{a}\,]\,] \gg [\, \underline{a} \overset{\mathrm{p}}{=} \underline{a}\,]\,]\,] \rceil, \mathrm{p}_0, \mathrm{c})]$

$[\mathrm{Mendelson}\ \mathbf{3.2}\ \mathrm{a} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a}\colon [\, \underline{a} \overset{\mathrm{p}}{=} \underline{a}\,]\,]$

$[\mathrm{Mendelson}\ \mathbf{3.2}\ \mathrm{a} \overset{\mathrm{tex}}{\rightarrow} \text{“Mendelson\textbackslash\ \textbackslash textbf\{3.2\}\textbackslash\ a”}]$

$[\mathrm{Mendelson}\ \mathbf{3.2}\ \mathrm{a} \overset{\mathrm{pyk}}{\rightarrow} \text{“mendelson proposition three two a”}]$

Mendelson **3.2** b

[Mendelson **3.2** b $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ S1'\ \gg\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \overset{p}{=} \underline{a}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{Mendelson } \mathbf{3.2}\ a \gg\ [\ \underline{a} \overset{p}{=} \underline{a}\ ]\ ]\ ;\ [\ [\ [\ \text{Mendelson } \mathbf{1.10}\ b \rhd\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \overset{p}{=} \underline{a}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ \rhd\ [\ \underline{a} \overset{p}{=} \underline{a}\ ]\ ]\ \gg\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ]\ ]\rfloor, p_0, c)$]

[Mendelson **3.2** b $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]$

[Mendelson **3.2** b $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{3.2}\ b"]

[Mendelson **3.2** b $\overset{\text{pyk}}{\longrightarrow}$ "mendelson proposition three two b"]

Mendelson **3.2** b_i

[Mendelson **3.2** b_i $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \vdash\ [\ [\ \text{Mendelson } \mathbf{3.2}\ b \gg\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ ;\ [\ [\ [\ MP' \rhd\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ \rhd\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ]\ \gg\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ ]\rfloor, p_0, c)$]

[Mendelson **3.2** b_i $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \vdash\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]$

[Mendelson **3.2** b_i $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{3.2}\ b_{i}"]

[Mendelson **3.2** b_i $\overset{\text{pyk}}{\longrightarrow}$ "inference mendelson proposition three two b"]

Mendelson **3.2** c

[Mendelson **3.2** c $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ S1'\ \gg\ [\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{Mendelson } \mathbf{3.2}\ b \gg\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{Mendelson } \mathbf{1.10}\ a \rhd\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ ]\ ]\ \rhd\ [\ [\ \underline{b} \overset{p}{=} \underline{a}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ \gg\ [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\rfloor, p_0, c)$]

[Mendelson **3.2** c $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$]

[Mendelson **3.2** c $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{3.2}\ c"]

[Mendelson **3.2** c $\overset{\text{pyk}}{\longrightarrow}$ "mendelson proposition three two c"]

Mendelson **3.2** c_{ii}

[Mendelson **3.2** c_{ii} $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \vdash [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \vdash [$ Mendelson **3.2** c $\gg [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]$; $[\ [\ [\ [$ $MP'_d \rhd [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \rhd [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ] \rhd [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ] \gg [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ \rfloor, p_0, c)]$

[Mendelson **3.2** c_{ii} $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \vdash [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \vdash [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$]

[Mendelson **3.2** c_{ii} $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{3.2}\ c_{ii}"]

[Mendelson **3.2** c_{ii} $\overset{\text{pyk}}{\longrightarrow}$ "inference inference mendelson proposition three two c"]

Mendelson **3.2** c_{iih}

[Mendelson **3.2** c_{iih} $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ] \vdash [$ $[\ \underline{h} \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ] \vdash [\ [$ Mendelson **1.8** $\gg [\ \underline{h} \Rightarrow \underline{h}\ ]\ ]$; $[\ [$ Mendelson **3.2** c $\gg [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]$; $[\ [\ [$ $A1'_i \rhd [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]$; $[\ [\ [\ [\ [$ $A2'_{iid} \rhd [\ \underline{h} \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$; $[\ [$ Mendelson $\mathbf{1.8}_i \rhd [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rfloor, p_0, c)]$

[Mendelson **3.2** c_{iih} $\overset{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ] \vdash [\ [\ \underline{h} \Rightarrow [$ $\underline{b} \overset{p}{=} \underline{c}\ ]\ ] \vdash [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]$]

[Mendelson **3.2** c_{iih} $\overset{\text{tex}}{\longrightarrow}$ "Mendelson\ \textbf{3.2}\ c_{iih}"]

[Mendelson **3.2** c_{iih} $\overset{\text{pyk}}{\longrightarrow}$ "hypothetical inference inference mendelson proposition three two c"]

Mendelson **3.2** d

[Mendelson **3.2** d $\overset{\text{proof}}{\longrightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [$ Mendelson **3.2** c $\gg [\ [$ $\underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]$; $[\ [\ [$ Mendelson **1.47** c $\rhd [\ [\ \underline{a} \overset{p}{=} \underline{b}$ $\ ] \Rightarrow [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]$ $\ ]\ ]\ ]$; $[\ [$ Mendelson **3.2** b $\gg [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ]$; $[\ [\ [\ [$ Mendelson **1.10** a $\rhd [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{b} \overset{p}{=} \underline{c}\ ]\ ]\ ] \rhd [\ [\ \underline{b} \overset{p}{=} \underline{c}\ ] \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ]$ $\Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]$; $[\ [$ Mendelson **1.47** c $\rhd [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \gg [\ [$

$\underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ ], p_0, c)]$

[Mendelson **3.2** d $\overset{stmt}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$]

[Mendelson **3.2** d $\overset{tex}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ d"]

[Mendelson **3.2** d $\overset{pyk}{\rightarrow}$ "mendelson proposition three two d"]

Mendelson **3.2** d_{ii}

[Mendelson **3.2** d_{ii} $\overset{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \vdash [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \vdash [$ [Mendelson **3.2** d $\gg [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$; $[\ [\ [\ [$ $MP'_d \rhd [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \rhd [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ] \rhd [\ \underline{c} \overset{p}{=} \underline{b}\ ]$ $] \gg [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\ \rfloor, p_0, c)]$

[Mendelson **3.2** d_{ii} $\overset{stmt}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \vdash [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \vdash [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$]

[Mendelson **3.2** d_{ii} $\overset{tex}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ d_{ii}"]

[Mendelson **3.2** d_{ii} $\overset{pyk}{\rightarrow}$ "inference inference mendelson proposition three two d"]

Mendelson **3.2** d_{iih}

[Mendelson **3.2** d_{iih} $\overset{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ] \vdash [\ [\ \underline{h} \Rightarrow [\ \underline{c} \overset{p}{=} \underline{b}\ ]\ ] \vdash [\ [$ Mendelson **1.8** $\gg [\ \underline{h} \Rightarrow \underline{h}\ ]\ ]$; $[\ [$ Mendelson **3.2** d $\gg [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$; $[\ [\ [$ $A1'_i \rhd [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [$ $\underline{c} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]$; $[\ [\ [\ [\ [$ $A2'_{iid} \rhd [\ \underline{h} \Rightarrow [\ [\ \underline{a} \overset{p}{=} \underline{b}\ ] \Rightarrow [\ [\ \underline{c} \overset{p}{=} \underline{b}\ ]$ $] \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ]\ ] \rhd [\ \underline{h} \Rightarrow [\ \underline{c} \overset{p}{=} \underline{b}\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow$ $[\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]$; $[\ [$ Mendelson $\mathbf{1.8}_i \rhd [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]$ $]\ ]\ ]\ ]\ ]\ ]\ \rfloor, p_0, c)]$

[Mendelson **3.2** d_{iih} $\overset{stmt}{\rightarrow}$ $S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{b}\ ]\ ] \vdash [\ [\ \underline{h} \Rightarrow [$ $\underline{c} \overset{p}{=} \underline{b}\ ]\ ] \vdash [\ \underline{h} \Rightarrow [\ \underline{a} \overset{p}{=} \underline{c}\ ]\ ]\ ]\ ]$]

[Mendelson **3.2** d_{iih} $\overset{tex}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ d_{iih}"]

[Mendelson **3.2** d_{iih} $\overset{pyk}{\rightarrow}$ "hypothetical inference inference mendelson proposition three two d"]

Mendelson **3.2** f

[Mendelson **3.2** f $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash$ [[Mendelson **3.2** f i $\gg$ [$\dot{0} \overset{p}{=}$ [$\dot{0} \dot{+} \dot{0}$]]] ; [[Mendelson **3.2** f ii $\gg$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}'$]]]]] ; [[[Induction $\rhd$ [$\dot{0} \overset{p}{=}$ [$\dot{0} \dot{+} \dot{0}$]]] $\rhd$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}'$]]]]] $\gg$ [$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]]]]] $\rfloor, p_0, c)$]

[Mendelson **3.2** f $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash$ [$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]]]

[Mendelson **3.2** f $\overset{\text{tex}}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ f"]

[Mendelson **3.2** f $\overset{\text{pyk}}{\rightarrow}$ "mendelson proposition three two f"]

Mendelson **3.2** f i

[Mendelson **3.2** f i $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash$ [[S5′ $\gg$ [[$\dot{0} \dot{+} \dot{0}$] $\overset{p}{=} \dot{0}$]] ; [[Mendelson **3.2** $b_i \rhd$ [[$\dot{0} \dot{+} \dot{0}$] $\overset{p}{=} \dot{0}$]] $\gg$ [$\dot{0} \overset{p}{=}$ [$\dot{0} \dot{+} \dot{0}$]]]]] $\rfloor, p_0, c)$]

[Mendelson **3.2** f i $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash$ [$\dot{0} \overset{p}{=}$ [$\dot{0} \dot{+} \dot{0}$]]]

[Mendelson **3.2** f i $\overset{\text{tex}}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ f\ i"]

[Mendelson **3.2** f i $\overset{\text{pyk}}{\rightarrow}$ "mendelson proposition three two f i"]

Mendelson **3.2** f ii

[Mendelson **3.2** f ii $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash$ [[Mendelson **1.8** $\gg$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]]]] ; [[S6′ $\gg$ [[$\dot{0} \dot{+}$ [$\dot{x}'$]] $\overset{p}{=}$ [[$\dot{0} \dot{+}$ [$\dot{x}$]] ′]]]] ; [[[$A1'_i \rhd$ [[$\dot{0} \dot{+}$ [$\dot{x}'$]] $\overset{p}{=}$ [[$\dot{0} \dot{+}$ [$\dot{x}$]] ′]]] $\gg$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [[$\dot{0} \dot{+}$ [$\dot{x}'$]] $\overset{p}{=}$ [[$\dot{0} \dot{+}$ [$\dot{x}$]] ′]]]] ; [[[$S2'_{ih} \rhd$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]]]] $\gg$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [[$\dot{0} \dot{+}$ [$\dot{x}$]] ′]]]] ; [[[[Mendelson **3.2** $d_{iih} \rhd$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [[$\dot{0} \dot{+}$ [$\dot{x}$]] ′]]]] $\rhd$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [[$\dot{0} \dot{+}$ [$\dot{x}'$]] $\overset{p}{=}$ [[$\dot{0} \dot{+}$ [$\dot{x}$]] ′]]]] $\gg$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}'$]]]]]] ; [[Mendelson $\mathbf{1.8}_i \rhd$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}'$]]]]] $\gg$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}'$]]]]]]]]]]]] $\rfloor, p_0, c)$]

[Mendelson **3.2** f ii $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash$ [[$\dot{x} \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}$]]] $\Rightarrow$ [$\dot{x}' \overset{p}{=}$ [$\dot{0} \dot{+}$ [$\dot{x}'$]]]]]

[Mendelson **3.2** f ii $\overset{\text{tex}}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ f\ ii"]

[Mendelson **3.2** f ii $\overset{\text{pyk}}{\rightarrow}$ "mendelson proposition three two f ii"]

Mendelson **3.2** g

[Mendelson **3.2** g $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash$ [[Mendelson **3.2** g i $\gg$ [[$\dot{\mathrm{y}}' \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $'$]]] ; [[Mendelson **3.2** g ii $\gg$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}'$]] $'$]]]] ; [[[Induction $\triangleright$ [[$\dot{\mathrm{y}}' \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $'$]]] $\triangleright$ [[[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}'$]] $'$]]]] $\gg$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]]]] $\rceil, \mathrm{p}_0, \mathsf{c})$]

[Mendelson **3.2** g $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]]

[Mendelson **3.2** g $\overset{\text{tex}}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ g"]

[Mendelson **3.2** g $\overset{\text{pyk}}{\rightarrow}$ "mendelson proposition three two g"]

Mendelson **3.2** g i

[Mendelson **3.2** g i $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash$ [[S5′ $\gg$ [[$\dot{\mathrm{y}}' \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [$\dot{\mathrm{y}}'$]]] ; [[S5′ $\gg$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [$\dot{\mathrm{y}}$]]] ; [[[$\mathrm{S2}'_{\mathrm{i}} \triangleright$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [$\dot{\mathrm{y}}$]]] $\gg$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $' \overset{\mathrm{p}}{=}$ [$\dot{\mathrm{y}}'$]]] ; [[[Mendelson **3.2** $\mathrm{d}_{\mathrm{ii}} \triangleright$ [[$\dot{\mathrm{y}}' \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [$\dot{\mathrm{y}}'$]]] $\triangleright$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $' \overset{\mathrm{p}}{=}$ [$\dot{\mathrm{y}}'$]]] $\gg$ [[$\dot{\mathrm{y}}' \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $'$]]]]]] $\rceil, \mathrm{p}_0, \mathsf{c})$]

[Mendelson **3.2** g i $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash$ [[$\dot{\mathrm{y}}' \dot{+} \dot{0}$] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+} \dot{0}$] $'$]]]

[Mendelson **3.2** g i $\overset{\text{tex}}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ g\ i"]

[Mendelson **3.2** g i $\overset{\text{pyk}}{\rightarrow}$ "mendelson proposition three two g i"]

Mendelson **3.2** g ii

[Mendelson **3.2** g ii $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash$ [[Mendelson **1.8** $\gg$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]]] ; [[S6′ $\gg$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]] ; [[[$\mathrm{A1}'_{\mathrm{i}} \triangleright$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]] $\gg$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}'$]] $'$]]]] ; [[[$\mathrm{S2}'_{\mathrm{ih}} \triangleright$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]]] $\gg$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $' \overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $''$]]]] ; [[[[Mendelson **3.2** $\mathrm{c}_{\mathrm{iih}} \triangleright$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]]] $\triangleright$ [[[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $' \overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $''$]]]] $\gg$ [[[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]] $\Rightarrow$ [[$\dot{\mathrm{y}}' \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $''$]]]] ; [[S6′ $\gg$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}$]] $'$]]] ; [[[$\mathrm{A1}'_{\mathrm{i}} \triangleright$ [[$\dot{\mathrm{y}} \dot{+}$ [$\dot{\mathrm{x}}'$]] $\overset{\mathrm{p}}{=}$ [[$\dot{\mathrm{y}} \dot{+}$

$[\dot{x}]]']]] \gg [[[\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']] \dot{\Rightarrow} [[\dot{y} \dot{+} [\dot{x}']] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]; [[[S2'_{ih} \rhd [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}\dot{+}[\dot{x}']] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']]]]] \gg [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}\dot{+}[\dot{x}']]' \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]'']]]]; [[[[\text{Mendelson } \mathbf{3.2}\ d_{iih} \rhd [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}'\dot{+}[\dot{x}']] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]'']]]]] \rhd [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}\dot{+}[\dot{x}']]' \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]'']]]]] \gg [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}'\dot{+}[\dot{x}']] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}']]']]]]]; [[\text{Mendelson } \mathbf{1.8}_i \rhd [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}'\dot{+}[\dot{x}']] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}']]']]]] \gg [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}'\dot{+}[\dot{x}']] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}']]']]]]]]]]]]]]]] \urcorner, p_0, c)]$

$[\text{Mendelson } \mathbf{3.2}\ g\ ii \stackrel{stmt}{\rightarrow} S' \vdash [[[\dot{y}'\dot{+}[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}]]']] \dot{\Rightarrow} [[\dot{y}'\dot{+}[\dot{x}']] \stackrel{p}{=} [[\dot{y}\dot{+}[\dot{x}']]']]]]]$

$[\text{Mendelson } \mathbf{3.2}\ g\ ii \stackrel{tex}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ g\ ii"]

$[\text{Mendelson } \mathbf{3.2}\ g\ ii \stackrel{pyk}{\rightarrow}$ "mendelson proposition three two g ii"]

Mendelson **3.2** h

$[\text{Mendelson } \mathbf{3.2}\ h \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\ulcorner S' \vdash [[\text{Mendelson } \mathbf{3.2}\ h\ i \gg [[\dot{x}\dot{+}\dot{0}] \stackrel{p}{=} [\dot{0}\dot{+}[\dot{x}]]]]]; [[\text{Mendelson } \mathbf{3.2}\ h\ ii \gg [[[\dot{x}\dot{+}[\dot{y}]] \stackrel{p}{=} [\dot{y}\dot{+}[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dot{+}[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dot{+}[\dot{x}]]]]]]; [[[\text{Induction} \rhd [[\dot{x}\dot{+}\dot{0}] \stackrel{p}{=} [\dot{0}\dot{+}[\dot{x}]]]]] \rhd [[[\dot{x}\dot{+}[\dot{y}]] \stackrel{p}{=} [\dot{y}\dot{+}[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dot{+}[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dot{+}[\dot{x}]]]]]]] \gg [[\dot{x}\dot{+}[\dot{y}]] \stackrel{p}{=} [\dot{y}\dot{+}[\dot{x}]]]]]]] \urcorner, p_0, c)]$

$[\text{Mendelson } \mathbf{3.2}\ h \stackrel{stmt}{\rightarrow} S' \vdash [[\dot{x}\dot{+}[\dot{y}]] \stackrel{p}{=} [\dot{y}\dot{+}[\dot{x}]]]]]$

$[\text{Mendelson } \mathbf{3.2}\ h \stackrel{tex}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ h"]

$[\text{Mendelson } \mathbf{3.2}\ h \stackrel{pyk}{\rightarrow}$ "mendelson proposition three two h"]

Mendelson **3.2** h i

$[\text{Mendelson } \mathbf{3.2}\ h\ i \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\ulcorner S' \vdash [[S5' \gg [[\dot{x}\dot{+}\dot{0}] \stackrel{p}{=} [\dot{x}]]]]; [[\text{Mendelson } \mathbf{3.2}\ f \gg [\dot{x} \stackrel{p}{=} [\dot{0}\dot{+}[\dot{x}]]]]]; [[[\text{Mendelson } \mathbf{3.2}\ c_{ii} \rhd [[\dot{x}\dot{+}\dot{0}] \stackrel{p}{=} [\dot{x}]]]] \rhd [\dot{x} \stackrel{p}{=} [\dot{0}\dot{+}[\dot{x}]]]]] \gg [[\dot{x}\dot{+}\dot{0}] \stackrel{p}{=} [\dot{0}\dot{+}[\dot{x}]]]]]] \urcorner, p_0, c)]$

$[\text{Mendelson } \mathbf{3.2}\ h\ i \stackrel{stmt}{\rightarrow} S' \vdash [[\dot{x}\dot{+}\dot{0}] \stackrel{p}{=} [\dot{0}\dot{+}[\dot{x}]]]]]$

$[\text{Mendelson } \mathbf{3.2}\ h\ i \stackrel{tex}{\rightarrow}$ "Mendelson\ \textbf{3.2}\ h\ i"]

[Mendelson **3.2** h i $\stackrel{\text{pyk}}{\to}$ "mendelson proposition three two h i"]

Mendelson **3.2** h ii

[Mendelson **3.2** h ii $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ \text{Mendelson } \mathbf{1.8} \gg [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ]$
$\stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ S6' \gg [\ [\ \dot{x} \dotplus$
$[\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ]'\ ]\ ]\ ]\ ; [\ [\ [\ A1'_i \rhd [\ [\ \dot{x} \dotplus [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ]$
$'\ ]\ ]\ ] \gg [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dotplus [\ \dot{y}$
$]\ ]'\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ \text{Mendelson } \mathbf{3.2} \text{ g} \gg [\ [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ]$
$; [\ [\ [\ A1'_i \rhd [\ [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ] \gg [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=}$
$[\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ S2'_{ih} \rhd [\ [$
$[\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ] \gg [\ [$
$[\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ]' \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ;$
$[\ [\ [\ [\ \text{Mendelson } \mathbf{3.2} \text{ c}_{iih} \rhd [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [$
$\dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ]'\ ]\ ]\ ]\ ] \rhd [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [$
$\dot{x} \dotplus [\ \dot{y}\ ]\ ]' \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ]\ ]\ ] \gg [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ]$
$\dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{Mendelson } \mathbf{3.2} \text{ d}_{iih} \rhd [$
$[\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]\ ]\ ]$
$\rhd [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]'\ ]\ ]$
$]\ ] \gg [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ]$
$]\ ]\ ]\ ; [\ [\ \text{Mendelson } \mathbf{1.8}_i \rhd [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [$
$\dot{y}'\ ]\ ] \stackrel{p}{=} [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ] \gg [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus$
$[\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rceil, p_0, c)$]

[Mendelson **3.2** h ii $\stackrel{\text{stmt}}{\to}$ $S' \vdash [\ [\ [\ \dot{x} \dotplus [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ \dot{y} \dotplus [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dotplus [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ \dot{y}' \dotplus [\ \dot{x}\ ]\ ]\ ]\ ]\ ]$]

[Mendelson **3.2** h ii $\stackrel{\text{tex}}{\to}$ "Mendelson\ \textbf{3.2}\ h\ ii"]

[Mendelson **3.2** h ii $\stackrel{\text{pyk}}{\to}$ "mendelson proposition three two h ii"]

hyp

[hyp $\stackrel{\text{tex}}{\to}$ "\mathsf{hyp}"]

[hyp $\stackrel{\text{pyk}}{\to}$ "hypothesis"]

instance

[instance $\stackrel{\text{tex}}{\to}$ "\mathsf{instance}"]

[$\mathsf{instance} \overset{\text{pyk}}{\to}$ “instance”]

conclusion

[$\mathsf{conclusion} \overset{\text{tex}}{\to}$ “\mathsf{conclusion}”]

[$\mathsf{conclusion} \overset{\text{pyk}}{\to}$ “conclusion”]

$* \trianglerighteq *$

[$\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq \mathsf{y} \doteqdot [\ \mathrm{MP}' \rhd \mathsf{x}\ ] \rhd \mathsf{y}]\rceil)$]

[$\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{tex}}{\to}$ “#1.
\unrhd #2.”]

[$\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{pyk}}{\to}$ “$*$ macro modus ponens $*$”]

$* \trianglerighteq * \trianglerighteq *$

[$\mathsf{x} \trianglerighteq \mathsf{y} \trianglerighteq \mathsf{z} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq \mathsf{y} \trianglerighteq \mathsf{z} \doteqdot [\ [\ \mathrm{MP}'_{\mathrm{d}} \rhd \mathsf{x}\ ] \rhd \mathsf{y}\ ] \rhd \mathsf{z}]\rceil)$]

[$\mathsf{x} \trianglerighteq \mathsf{y} \trianglerighteq \mathsf{z} \overset{\text{tex}}{\to}$ “#1.
\unrhd #2.
\unrhd #3.”]

[$\mathsf{x} \trianglerighteq \mathsf{y} \trianglerighteq \mathsf{z} \overset{\text{pyk}}{\to}$ “$*$ macro first modus ponens $*$ macro second modus ponens $*$”]

$* \trianglerighteq_{\mathrm{h}} *$

[$\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \doteqdot [\ \mathrm{MP}'_{\mathrm{h}} \rhd \mathsf{x}\ ] \rhd \mathsf{y}]\rceil)$]

[$\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \overset{\text{tex}}{\to}$ “#1.
\unrhd_h #2.”]

[$\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \overset{\text{pyk}}{\to}$ “$*$ hypothetical macro modus ponens $*$”]

$* \trianglerighteq_{\mathrm{h}} * \trianglerighteq_{\mathrm{h}} *$

[$\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \trianglerighteq_{\mathrm{h}} \mathsf{z} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \trianglerighteq_{\mathrm{h}} \mathsf{z} \doteqdot [\ [\ \mathrm{MP}'_{\mathrm{hd}} \rhd \mathsf{x}\ ] \rhd \mathsf{y}\ ] \rhd \mathsf{z}]\rceil)$]

[$\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \trianglerighteq_{\mathrm{h}} \mathsf{z} \overset{\text{tex}}{\to}$ “#1.
\unrhd_h #2.

\unrhd_h #3."]

[$\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \trianglerighteq_{\mathrm{h}} \mathsf{z} \overset{\mathrm{pyk}}{\rightarrow}$ "∗ hypothetical macro first modus ponens ∗ hypothetical macro second modus ponens var ∗"]

Line $* : \bullet \;\; * \trianglerighteq * \gg *; *$

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{name}}{\rightarrow}$ "
Line \, #1.
:\bullet{}\ #2.
\unrhd{} #3.
\gg #4.
; #5."]

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \doteqdot (\, [\, [\, \mathsf{u} \trianglerighteq_{\mathrm{h}} \mathsf{v}\,] \;\gg\; [\, \mathsf{hyp} \dot{\Rightarrow} \mathsf{i}\,]\,]\, ; \mathbf{let}\ \mathsf{l} \doteqdot \mathsf{hyp} \dot{\Rightarrow} \mathsf{i}\ \ \mathrm{in}\ \ \mathsf{p})] \rceil)]$

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{tex}}{\rightarrow}$ "
\newline \makebox [0.1\textwidth]{}%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}$#2.
\unrhd{} #3.
{}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#4.
$\hfill \makebox [0mm][l]{\quad ; }}#5."]

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{pyk}}{\rightarrow}$ "line ∗ hypothesis modus ponens ∗ modus ponens ∗ indeed ∗ end line ∗"]

Line $* : \bullet \;\; * \circ \trianglerighteq * \gg *; *$

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \circ \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{name}}{\rightarrow}$ "
Line \, #1.
:\bullet{}\ #2.
\circ{} \unrhd{} #3.
\gg #4.
; #5."]

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \circ \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \circ \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \doteqdot (\, [\, [\, [\, \text{Mendelson } \mathbf{1.47}\ \mathsf{b} \triangleright \mathsf{v}\,] \triangleright \mathsf{u}\,] \;\gg\; [\, \mathsf{hyp} \dot{\Rightarrow} \mathsf{i}\,]\,]\, ; \mathbf{let}\ \mathsf{l} \doteqdot \mathsf{hyp} \dot{\Rightarrow} \mathsf{i}\ \ \mathrm{in}\ \ \mathsf{p})] \rceil)]$

[$\mathsf{Line}\, \mathsf{l} : \bullet\, \mathsf{u} \circ \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\mathrm{tex}}{\rightarrow}$ "
\newline \makebox [0.1\textwidth]{}%

\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}$ #2.
\circ{} \unrhd{} #3.
{}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#4.
$\hfill \makebox [0mm][l]{\quad ;}}#5."]

[Line $\mathsf{l} : \bullet\ \mathsf{u} \circ \trianglerighteq \mathsf{v} \gg \mathsf{i}; \mathsf{p} \overset{\text{pyk}}{\rightarrow}$ "line $*$ hypothesis first modus ponens $*$ modus ponens $*$ indeed $*$ end line $*$"]

Line $* : \bullet\ \ * \trianglerighteq * \circ \gg *; *$

[Line $\mathsf{l} : \bullet\ \mathsf{u} \trianglerighteq \mathsf{v} \circ\ \gg \mathsf{i}; \mathsf{p} \overset{\text{name}}{\rightarrow}$ "
Line \, #1.
:\bullet{}\ #2.
\unrhd{} #3.
\circ{} \gg #4.
; #5."]

[Line $\mathsf{l} : \bullet\ \mathsf{u} \trianglerighteq \mathsf{v} \circ\ \gg \mathsf{i}; \mathsf{p} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil$[Line $\mathsf{l} : \bullet\ \mathsf{u} \trianglerighteq \mathsf{v} \circ\ \gg \mathsf{i}; \mathsf{p} \doteqdot ($ [[[Mendelson **1.47** $\mathsf{e} \triangleright \mathsf{u}$] $\triangleright \mathsf{v}$] $\gg$ [$\mathsf{hyp} \Rightarrow \mathsf{i}$]] ; **let** $\mathsf{l} \doteqdot \mathsf{hyp} \Rightarrow \mathsf{i}$ in p)]$\rceil$)]

[Line $\mathsf{l} : \bullet\ \mathsf{u} \trianglerighteq \mathsf{v} \circ\ \gg \mathsf{i}; \mathsf{p} \overset{\text{tex}}{\rightarrow}$ "
\newline \makebox [0.1\textwidth]{}%
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\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}$ #2.
\unrhd{} #3.
\circ{} {}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#4.
$\hfill \makebox [0mm][l]{\quad ;}}#5."]

[Line $\mathsf{l} : \bullet\ \mathsf{u} \trianglerighteq \mathsf{v} \circ\ \gg \mathsf{i}; \mathsf{p} \overset{\text{pyk}}{\rightarrow}$ "line $*$ hypothesis second modus ponens $*$ modus ponens $*$ indeed $*$ end line $*$"]

Line $* : \bullet\ \ * \trianglerighteq * \trianglerighteq * \gg *; *$

[Line $\mathsf{l} : \bullet\ \mathsf{u} \trianglerighteq \mathsf{v} \trianglerighteq \mathsf{z} \gg \mathsf{i}; \mathsf{p} \overset{\text{name}}{\rightarrow}$ "
Line \, #1.
:\bullet{}\ #2.
\unrhd{} #3.

\unrhd{} #4.
\gg #5.
; #6.”]

[Line l : • u ⊵ v ⊵ z ≫ i; p $\overset{\text{macro}}{\to}$ λt.λs.λc.$\tilde{\mathcal{M}}_4$(t, s, c, ⌈[Line l : • u ⊵ v ⊵ z ≫ i; p ≝ ([[u $\trianglerighteq_{\text{h}}$ v $\trianglerighteq_{\text{h}}$ z] ≫ [hyp ⇛ i]] ; **let** l ≝ hyp ⇛ i in p)]⌉)]

[Line l : • u ⊵ v ⊵ z ≫ i; p $\overset{\text{tex}}{\to}$ “
\newline \makebox [0.1\textwidth]{}%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}$ #2.
\unrhd{} #3.
\unrhd{} #4.
{}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#5.
$\hfill \makebox [0mm][l]{\quad ;}}#6.”]

[Line l : • u ⊵ v ⊵ z ≫ i; p $\overset{\text{pyk}}{\to}$ “line ∗ hypothesis double modus ponens ∗ modus ponens ∗ modus ponens ∗ indeed ∗ end line ∗”]

Line ∗ : • Hypothesis ≫ ∗; ∗

[Line l : • Hypothesis ≫ i; p $\overset{\text{name}}{\to}$ “
Line \, #1.
:\bullet{}\ Hypothesis
\gg #2.
; #3.”]

[Line l : • Hypothesis ≫ i; p $\overset{\text{macro}}{\to}$ λt.λs.λc.$\tilde{\mathcal{M}}_4$(t, s, c, ⌈[Line l : • Hypothesis ≫ i; p ≝ ([Mendelson **1.8** ≫ [i ⇛ i]] ; **let** hyp ≝ i in **let** l ≝ i ⇛ i in p)]⌉)]

[Line l : • Hypothesis ≫ i; p $\overset{\text{tex}}{\to}$ “
\newline \makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}\makebox [0.4\textwidth][l]{$Hypothesis{}
\gg{}$}\quad
\parbox [t]{0.4\textwidth }{$#2.
$\hfill \makebox [0mm][l]{\quad ;}}#3.”]

[Line l : • Hypothesis ≫ i; p $\overset{\text{pyk}}{\to}$ “line ∗ hypothesis indeed ∗ end line ∗”]

Line $* : \bullet * \gg *; *$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{name}}{\rightarrow}$ “
Line \, #1.
:\bullet{}\ #2.
\gg #3.
; #4.”$]$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \doteqdot (\ [\ \mathsf{a} \gg \mathsf{i}\ ]\ ;\ [\ [\ [\ \mathsf{Hypothesize} \rhd \mathsf{i}\ ]\ \gg\ [\ \mathsf{hyp} \Rightarrow \mathsf{i}\ ]\ ]\ ; \mathbf{let}\ \mathsf{l} \doteqdot \mathsf{hyp} \Rightarrow \mathsf{i}\ \ \mathsf{in}\ \ \mathsf{p}\ ]\ )]\rceil)]$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{tex}}{\rightarrow}$ “
\newline \makebox [0.1\textwidth]{}%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}$#2.
{}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#3.
$\hfill \makebox [0mm][l]{\quad ;}}#4.”$]$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{pyk}}{\rightarrow}$ “line $*$ hypothesis because $*$ indeed $*$ end line $*$”$]$

Line $* : \bullet * \gg *; *$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{name}}{\rightarrow}$ “
Line \, #1.
:\bullet{}\ #2.
\gg #3.
; #4.”$]$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \doteqdot (\ [\ \mathsf{a} \gg\ [\ \mathsf{hyp} \Rightarrow \mathsf{i}\ ]\ ]\ ; \mathbf{let}\ \mathsf{l} \doteqdot \mathsf{hyp} \Rightarrow \mathsf{i}\ \ \mathsf{in}\ \ \mathsf{p})]\rceil)]$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{tex}}{\rightarrow}$ “
\newline \makebox [0.1\textwidth]{}%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$#1.
$:$\bullet$}$#2.
{}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#3.
$\hfill \makebox [0mm][l]{\quad ;}}#4.”$]$

$[\mathsf{Line}\,\mathsf{l} : \bullet\ \mathsf{a} \gg \mathsf{i}; \mathsf{p} \overset{\text{pyk}}{\rightarrow}$ “line $*$ hypothesis raw because $*$ indeed $*$ end line $*$”$]$