Ship Routing and Scheduling

Course: Optimization problems in production planning

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Overview

- Concepts - what is ship routing
- Details
  - Models
  - Solution methods
- Perspectives
Concepts

[Ronen 1994]:
Routing - Assignments of a sequence of ports to a ship.
Scheduling - When time is considered.

Three general modes:

1. Industrial
   - Cargo owner controls ships
   - Ship cargo at minimum cost

2. Tramp
   - Shipping company
   - Contract committed cargo
   - Optional cargo
   - Maximize profit of optional cargo

3. Liner
   - "Bus company"
   - Lines operated at given schedules
   - Supplies and demands of commodities given
   - Maximize revenue on network
Levels of Planning

Whenever shipping is part of the supply chain:

- Strategic – network design
- Tactical – fleet assignment, size and mix
- Operational – schedule ships, revenue management

Top level decisions are commonly done using best practice

Little optimization – huge impact.

Most scheduling does not take inventory at ports into account.

Problematic due to import/export levels, e.g. Asia to Europe
Maritime Networks

Constraints:

- Depends on fleet mix and size
- Ship size and other specifications
- Tide
- Fees for use of canals

Networks:

- Hub-and-spoke: Common for liners

![Hub-and-spoke network diagram]

- Direct lines: Alternative especially for industrial and tramp

![Direct lines network diagram]
Industrial

Problem:

1. Minimize cost of all ship transports
2. Lift all cargo from loading port to discharge port

About 40% of all literature propose Set Partition formulations.

Set Partition formulation:

- $N$ Commodity
- $V$ Ships
- $R_v$ Schedule of ship $v$
- $c_{vr}$ cost of schedule $r$ for ship $v$
- $a_{vr}$ ship $v$ transports commodity $i$ on schedule $r$

Spot charges can be used to transport surplus cargo.

\[
\begin{align*}
\min & \sum_{v \in V} \sum_{r \in R_v} c_{vr}x_{vr} + \sum_{i \in N} c_{SPOTi}s_i \\
\text{s.t.} & \sum_{v \in V} \sum_{r \in R_v} a_{ivr}x_{vr} + s_i = 1 & \forall i \in N \\
& \sum_{r \in R_v} x_{vr} = 1 & \forall v \in V \\
& s_i \in \{0, 1\} & \forall i \in N \\
& x_{vr} \in \{0, 1\} & \forall v \in V, \forall r \in R_v
\end{align*}
\]
Industrial – cont.

Schedules (columns) are found in a underlying network:

- Nodes are ports – pairs of loading (circle), discharge (square)
- Edges are shortest path between ports

Path in network is a schedule.

Corresponds to a Pickup and Delivery Problem.

Enumerate all routes on tightly constrained problems. Otherwise use column generation.
Tramp

Problem:

1. Maximize profit

2. *Contracts of affreightment* – obligated to carry cargo at given times

3. Resemble taxis – carry optional cargo at charge

Set Packing problem:

- $N_c$ committed cargo – $N_o$ optional cargo –
- $p_{vr}$ revenue of schedule $r$ for ship $v$ –

\[
\begin{align*}
\text{max} & \quad \sum_{v \in V} \sum_{r \in R_v} (p_{vr} - c_{vr})x_{vr} \\
\quad & + \sum_{i \in N_c} p_{\text{SPOT}i} s_i \\
\text{s.t.} & \quad \sum_{v \in V} \sum_{r \in R_v} a_{ivr} x_{vr} + s_i = 1 \quad \forall i \in N_c \\
& \quad \sum_{v \in V} \sum_{r \in R_v} a_{ivr} x_{vr} \leq 1 \quad \forall i \in N_o \\
& \quad \sum_{r \in R_v} x_{vr} = 1 \quad \forall v \in V \\
& \quad s_i \in \{0, 1\} \quad \forall i \in N_c \\
& \quad x_{vr} \in \{0, 1\} \quad \forall v \in V, \forall r \in R_v
\end{align*}
\]

Same solution methods as industrial shipping.
Liner

Significantly different than previous problems.

Strategic decisions:
1. Route and schedule design for liners
2. Fleet size and mix

Tactical decision:
3. Fleet deployment

Operational decision:
4. Cargo booking

1-3 decided before hand – top level decision.

Focus on cargo booking/revenue management

Problem:
1. Route commodities to maximize revenue
2. Capacitated network
3. Pairs of suppliers and demanders
Liner Network

- Nodes are ports
- Copies of nodes for each time period
- Ship scheduled in rotations
- Fleet assigned to overcome given rotations

Example network - supply (circle) and demand (square):
Liner Network – Rotations

Example network – rotation 1:

Example network – rotation 2:

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Liner Network – expanded in time

Time directed network.

All ports are connected with itself in the next time period. I.e., commodities can wait at ports before being shipped.

Rotations can be less than time period.

Several ships can contribute to edge capacity.

A supply/demand pair can be, e.g.

- from port 4 at time 2 to 1 arriving at time 2.
- from port 1 at time 1 to 3 arriving at time 3.
- from port 3 at time 2 to 4 arriving at time 2.
Multicommodity Flow Problem

A node-arc formulation:
- $K$ commodities – $V$ ports × time periods –
- $r_k$ profit of commodity $k$ –
- $c_{ij}$ cost for shipping $k$ between $i$ and $j$ –
- $O_k$ and $D_k$ is a supplier/delivery pair –
- $U_{ij}$ is the capacity of $(i, j)$ –

\[
\begin{align*}
\max & \sum_{k \in K} \sum_{j \in V} r_k^k x_{O_kj}^k - \sum_{k \in K} \sum_{i,j \in V} c_{ij}^k x_{ij}^k \\
\text{s.t.} & \sum_{j \in V} x_{ji}^k - \sum_{j \in V} x_{ij}^k \begin{cases} 
\leq d_k & i = O_k \\
\geq -d_k & i = D_k \\
= 0 & i \neq O_k, D_k
\end{cases} \quad \forall k \in K, \forall i \in V \\
& \sum_{k \in K} x_{ij}^k \leq U_{ij} \quad \forall i, j \in V \\
& x_{ij}^k \text{ is integer} \quad \forall i, j \in V, \forall k \in K
\end{align*}
\]
Dantzig-Wolfe decomposition

Path formulation:

\[
\begin{align*}
\max & \sum_{k \in K} \sum_{p \in P_k} (r^k - c_p)f(p) \\
\text{s.t.} & \sum_{k \in K} \sum_{p \in P_k} \delta_{ij}(p)f(p) \leq U_{ij} & \forall i, j \in V \\
& \sum_{p \in P_k} f(p) \leq d_k & \forall k \in K \\
x_{ij}^k & \text{ is integer} & \forall i, j \in V, \forall k \in K
\end{align*}
\]

Given duals: \( \pi \leq 0 \) of (13) and \( \sigma \leq 0 \) (14).

Negative reduced cost column is a path for \( k \) so

\[ \hat{c} = \sum_{i,j \in V} (c_{ij}^k - \pi_{ij})x_{ij} - r^k - \sigma^k \]

Where

\[ c_{ij}^k - \pi_{ij} \geq 0 \forall i, j \in V \]

Solve a shortest path problem:

\[
\begin{align*}
\min & \sum_{i,j \in V} (c_{ij}^k - \pi_{ij})x_{ij} \\
\text{s.t.} & \sum_{j \in V} x_{ji}^k - \sum_{j \in V} x_{ij} \begin{cases} 
\leq 1 & i = O_k \\
\geq -1 & i = D_k \\
= 0 & i \neq O_k, D_k
\end{cases} & \forall i \in V \\
x_{ij} & \text{ is integer} & \forall i, j \in V
\end{align*}
\]
Comments

Not solved to integrality – LP relaxation is good enough.

Issues:

- Assumes unlimited amount of containers in ports
- Container repositioning
  - Containers must be shipped to low import ports
- Substitution
  - Some commodities need special container types
  - Containers types can be substituted with certain other types
- Reshipping time
  - When a commodity must change ship to reach demand
Container Repositioning

Usually considered as a stand alone problem.

With inter-balancing constraints [Løfstedt]:
- \( e \in K \) is the empty containers –
- \( c_{LEAS_i} \) is the leasing cost of a container at \( i \) –

\[
\begin{align*}
\text{max} & \quad \sum_{k \in K \setminus \{e\}} \sum_{j \in V} r^k x^k_{O_k,j} - \sum_{k \in K} \sum_{i,j \in V} c^k_{ij} x^k_{ij} \\
& - \sum_{i \in V} c_{LEAS_i} l_i \\
\text{s.t.} & \quad \sum_{j \in V} x^k_{ji} - \sum_{j \in V} x^k_{ij} \begin{cases} 
\leq d_k & i = O_k \\
\geq -d_k & i = D_k \\
= 0 & i \neq O_k, D_k 
\end{cases} \forall k \in K \setminus \{e\}, \forall i \in V \\
& \quad \sum_{k \in K} x^k_{ij} \leq U_{ij} \quad \forall i, j \in V \\
& \quad \sum_{k \in K} \sum_{j \in V} (x^k_{ji} - x^k_{ij}) - l_i = 0 \quad \forall i \in V \\
& \quad l_i \text{ is integer} \quad \forall i \in V \\
& \quad x^k_{ij} \text{ is integer} \quad \forall i, j \in V, \forall k \in K
\end{align*}
\]

Can also be decomposed.

Pricing problem is still a shortest path problem.
Future

Little research has been done in maritime optimization.

General

• Network Design
• Fleet Assignment
• Inventory Management (Vendor Managed Inventory)

Special within Liners:

• Commodity Flow
• Container repositioning
• Reshipping Times