Production Planning Models

Objective
- Survey the structure of the planning system
- general principles of the planning procedure
- generic classes of production planning models

Overview
- Mathematical model of production planning models
- Generic MRP planning procedure
- Advanced planning systems (APS)
- Supply chain master planning problems

Modeling Elements
Production planning
(when to produce which items to meet demand)
- production lots or batches
- planning horizon
- resources (storable, not storable)
- production cost
- inventory cost
- customer service level

More complex models
- Products compete for capacity of shared resources
- Precedence constraints
- Backlogging
- Set-up times, start-up times, change-over times

Assumptions
- All data are known in advance (demand forecast)
- All data are deterministic (not stochastic)
- Structure of production chain is given

Uncapacitated Lot-Sizing Model (LS-U)
Single-item, single-level, uncapacitated lot-sizing model
- time horizon $t = 1, \ldots, n$
- $d_t$ demand
- production cost: fixed cost $q_t$ + marginal cost $p_t$
- $p_t$ unit production cost
- $q_t$ fixed production cost
- $h_t$ inventory cost

Decision variables
- $x_t$ production lot size in period $t$
- $y_t$ binary variable indicating production period $t$
- $s_t$ inventory at end of period $t$

Model
$$\min \sum_{t=1}^{n} p_t x_t + q_t y_t + h_t s_t$$

Subject to:
$$s_{t-1} + x_t = d_t + s_t \quad \forall t$$
$$s_0 = s_n = 0$$
$$x_t \leq M_t y_t \quad \forall t$$
$$x_t \in \mathbb{R}_+, s_t \in \mathbb{R}_+, y_t \in \{0, 1\}$$
Master Production Scheduling Model

Multi-item (single-level) capacitated lot-sizing model
- set of items
- short-term horizon
- each item: uncapacitated lot sizing
- production plan linked together through capacity restrictions

Definitions
- $i = 1, \ldots, m$ items to be produced
- $k = 1, \ldots, K$ set of shared resources
- $t = 1, \ldots, n$ time periods

Model

$$\min \sum_{i=1}^{m} \sum_{t=1}^{n} p_i^t x_i^t + q_i^t y_i^t + h_i^t s_i^t$$

Subject to:
- $s_{i-1}^t + x_i^t = d_i^t + s_i^t \quad \forall i, t$
- $s_i^0 = s_i^n = 0 \quad \forall i$
- $x_i^t \leq M_i^t y_i^t \quad \forall i, t$
- $\sum_{i=1}^{m} \alpha_i^k x_i^t + \sum_{i=1}^{m} \beta_i^k y_i^t \leq L_k^t \quad \forall i, k$
- $x_i^t \in \mathbb{R}_+, s_i^t \in \mathbb{R}_+, y_i^t \in \{0, 1\}$

Resources
- Storable
- Not storable

Material Requirement Planning Model

Multi-item multi-level capacitated lot-sizing model
- Optimize simultaneously production and purchase of all items
- from raw materials to finished products
- satisfy external demands
- satisfy internal demands
- short-term horizon

Bill of materials (BOM)
Material Requirement Planning Model

Model:
- \( D(i) \) set of direct successors of \( i \) (items consuming \( i \))
- \( D(i) = \emptyset \) for finished products
- \( r_{ij} \) amount of item \( i \) required to make one item \( j \)
- \( \gamma_i \) lead-time to produce or deliver a lot of \( i \)
I.e. \( x_i \) can be delivered at time \( t + \gamma_i \)

Model
\[
\min \sum_{i=1}^{m} \sum_{t=1}^{n} p_i t x_i t + q_i t y_i t + h_i t s_i t
\]
Subject to:
\[
s_{i-1} t + x_{i-1} t \cdot \gamma_i = \left( d_i t + \sum_{j \in D(i)} r_{ij} t x_{j} t \right) + s_{i} t \quad \forall i, t
\]
\[
x_{i} t \leq M_i s_{i} t \quad \forall i, t
\]
\[
\sum_{i=1}^{m} \alpha^k x_{i} t + \sum_{i=1}^{m} p^k y_{i} t \leq L^k_i
\]
\[
x_{i} t \in \mathbb{R}_+, s_{i} t \in \mathbb{R}_+, y_{i} t \in \{0, 1\}
\]

Generic flow conservation

Production policies

Make-to-stock
- independent demand must be in stock when the customer demand arrives
- (based on demand forecast)

Make-to-order
- some activities can still be performed after ordering of products
- delivery lead time is time promised for delivery
- enough raw material and semi-finished products to terminate on time
- “push” phase (based on forecast)
- “pull” phase (based on demand)

MRP model is used to solve make-to-stock or “push” in make-to-order

Capacity of Resources

<table>
<thead>
<tr>
<th>Resource Description</th>
<th>Gross Capacity (hours/day)</th>
<th>Productivity Factor</th>
<th>Usable Capacity (hours/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine S100</td>
<td>8</td>
<td>0.95</td>
<td>7.6</td>
</tr>
<tr>
<td>Forklift</td>
<td>8</td>
<td>0.85</td>
<td>6.8</td>
</tr>
<tr>
<td>Machine ASS</td>
<td>16</td>
<td>0.85</td>
<td>13.6</td>
</tr>
<tr>
<td>Machine PPP</td>
<td>8</td>
<td>0.95</td>
<td>7.6</td>
</tr>
</tbody>
</table>
Hachman and Leachman paper

Generalize to

- General production functions
- Time lags

"Lead time": unavoidable production time
"Output lag": e.g. drying of paint
"Input lag": e.g. transport, inspection before use

Advanced Planning Systems

Supply Chain Planning
(purchasing, manufacturing, warehousing, transportation)

Strategic planning
- Management of changes in production process
- Acquisition of resources over long-term horizon based on aggregated data

Tactical planning
- Resource allocation over medium-term planning (aggregated data)
- E.g. Material flows, Inventory, Capacity utilization, Maintenance planning,

Operational planning
- Planning and execution of production tasks
- E.g. production sequencing

Advanced Planning Systems

Supply Chain Planning Problems

- Strategic Network Design Problem
- Master Planning (match production and demand, remove redundant buffers)
- Demand Forecasting, Capacity Planning