

LP in Standard and Slack Forms

$$\begin{aligned} \max \quad & \sum_{j=1}^n c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j \leq b_i \quad \text{for } i=1,2,\dots,m \\ & x_j \geq 0 \quad \text{for } j=1,2,\dots,n \end{aligned}$$

$$\begin{aligned} \max \quad & \sum_{j=1}^n c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j + x_{n+i} = b_i \quad \text{for } i=1,2,\dots,m \\ & x_j \geq 0 \quad \text{for } j=1,2,\dots,n+m \end{aligned}$$

LP in Slack Form

$$\begin{aligned} \max \quad & \sum_{j=1}^n c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j + x_{n+i} = b_i \quad \text{for } i=1, 2, \dots, m \\ & x_j \geq 0 \quad \text{for } j=1, 2, \dots, n+m \end{aligned}$$

$$\begin{aligned} \max \quad z &= \sum_{j=1}^n c_j x_j \\ \text{s.t.} \quad x_{n+i} &= b_i - \sum_{j=1}^n a_{ij} x_j \quad \text{for } i=1, 2, \dots, m \\ x_j &\geq 0 \quad \text{for } j=1, 2, \dots, n+m \end{aligned}$$

$$\begin{aligned} z &= 0 + \sum_{j=1}^n c_j x_j \\ x_{n+i} &= b_i - \sum_{j=1}^n a_{ij} x_j \quad \text{for } i=1, 2, \dots, m \end{aligned}$$

SIMPLEX: 1. Pivoting

- Can x_1 be increased without violating feasibility? By how much?

$$\begin{aligned}z &= 0 + 3x_1 + x_2 + 2x_3 \\x_4 &= 30 - x_1 - x_2 - 3x_3 \\x_5 &= 24 - 2x_1 - 2x_2 - 5x_3 \\x_6 &= 36 - 4x_1 - x_2 - 2x_3\end{aligned}$$

- If x_1 is increased beyond 30 then x_4 becomes negative.
- If x_1 is increased beyond 12 then x_5 becomes negative.
- If x_1 is increased beyond 9 then x_6 becomes negative.
- Constraint defining x_6 is **binding**.

SIMPLEX: 1. Pivoting

$$\begin{aligned}
 z &= 0 + 3(9 - x_2/4 - x_3/2 - x_6/4) + x_2 + 2x_3 \\
 x_4 &= 30 - (9 - x_2/4 - x_3/2 - x_6/4) - x_2 - 3x_3 \\
 x_5 &= 24 - 2(9 - x_2/4 - x_3/2 - x_6/4) - 2x_2 - 5x_3 \\
 x_1 &= 9 - x_2/4 - x_3/2 - x_6/4
 \end{aligned}$$

$$\begin{aligned}
 z &= 27 + x_2/4 + x_3/2 - 3x_6/4 \\
 x_4 &= 21 - 3x_2/4 - 5x_3/2 + x_6/4 \\
 x_5 &= 6 - 3x_2/2 - 4x_3 + x_6/2 \\
 x_1 &= 9 - x_2/4 - x_3/2 - x_6/4
 \end{aligned}$$

New basic variables: $x_1=9$, $x_4=21$, $x_5=6$

New objective value $z = 27$

New feasible basic solution: (9,0,0,21,6,0)

SIMPLEX: 1. Pivoting

- So x_1 can be increased to 9 without losing feasibility. The feasible solution is $(9,0,0,21,6,0)$.
- Slack form is rewritten to an equivalent slack form with x_1, x_4, x_5 as basic variables and with $(9,0,0,21,6,0)$ being its feasible basic solution.
- This rewriting is called **pivoting**.
- Binding constraint defining x_6 is rewritten so that it has x_1 on its left-hand side.
- All occurrences of x_1 in other constraints and in the objective function are replaced by the right-hand side of the binding constraint.

Pivoting in General

- PIVOT(N, B, A, b, c, v, l, e)
 - Compute the coefficients of the bounding constraint so that the **entering** basic variable x_e is expressed as a linear combination of the other variables.

$$b_e = b_l / a_{le} \quad a_{ej} = a_{lj} / a_{le}, \quad \forall j \in N \setminus e \quad a_{el} = 1 / a_{le}$$

- Compute the coefficients of the remaining constraints and the objective function (by substituting x_e by the right-hand side of the rewritten binding equation).

$$b_i = b_i - a_{ie} b_e, \quad \forall i \in B \setminus l \quad a_{ij} = a_{ij} - a_{ie} a_{ej}, \quad \forall j \in N \setminus e \quad a_{il} = -a_{ie} a_{el}$$

$$v = v + c_e b_e \quad c_j = c_j - c_e a_{ej}, \quad \forall j \in N \setminus e \quad c_l = -c_e a_{el}$$

- Compute new sets of basic and nonbasic variables (remove x_e from N and add it to B , remove x_l from B and add it to N).

SIMPLEX

- SIMPLEX starts with a slack form corresponding to some feasible basic solution and iterates:
 - Select a nonbasic variable x_e with $c_e > 0$. If no such x_e exists, SIMPLEX terminates. We will show that the feasible basic solution is optimal.
 - Select a basic variable x_l that most severely limits the nonbasic variable x_e . Ties are broken arbitrarily. We will show that LP is unbounded if no such x_l exists.
 - Pivot.

SIMPLEX – Open Issues

- How to decide that LP is feasible?
- What to do if the initial basic solution is infeasible?
- How to select entering and leaving variables?
- How to decide that LP is unbounded?
- Does SIMPLEX terminate?
- Does it terminate with an optimal solution?

Termination

- SIMPLEX computes a feasible basic solution during each iteration.
- When does SIMPLEX terminate?
 - When all coefficients in the objective function are negative.
 - When it becomes obvious that LP is unbounded.

SIMPLEX: LP is Unbounded

- Can x_2 be increased without violating feasibility? By how much?

$$\begin{aligned}z &= 111/4 + x_2/16 - x_5/8 - 11x_6/16 \\x_4 &= 69/4 + 3x_2/16 + 5x_5/8 - x_6/16 \\x_3 &= 3/2 + 3x_2/8 - x_5/4 + x_6/8 \\x_1 &= 33/4 + x_2/16 + x_5/8 - 5x_6/16\end{aligned}$$

- If x_2 is increased then x_4 also increases.
- If x_2 is increased then x_3 also increases.
- If x_2 is increased then x_1 also increases.
- No constraint is binding; LP is unbounded.

Termination

- The number of basic solutions is finite: Number of basic variables is m . They are selected from among $m+n$ variables. This can be done in

$$\binom{m+n}{m} = \frac{(n+m)!}{n!m!}$$

ways

- Each basic solution has exactly one objective value. If the objective value increases at each iteration, we will eventually end up with a solution where the coefficients of the objective function are all negative (or we will realize that the LP is unbounded).
- Is it possible that the objective value does not change?

Degeneracy

$$\begin{aligned} z &= 0 + x_1 + x_2 + x_3 \\ x_4 &= 8 - x_1 - x_2 \\ x_5 &= x_2 - x_3 \end{aligned}$$

$$\begin{aligned} z &= 8 + x_3 - x_4 \\ x_1 &= 8 - x_2 - x_4 \\ x_5 &= x_2 - x_3 \end{aligned}$$

$$\begin{aligned} z &= 8 + x_2 - x_4 - x_5 \\ x_1 &= 8 - x_2 - x_4 \\ x_3 &= x_2 - x_5 \end{aligned}$$

Degeneracy

$$\begin{aligned}z &= 8 + x_2 - x_4 - x_5 \\x_1 &= 8 - x_2 - x_4 \\x_3 &= x_2 - x_5\end{aligned}$$

$$\begin{aligned}z &= 16 - x_1 - 2x_4 - x_5 \\x_2 &= 8 - x_1 - x_4 \\x_3 &= 8 - x_1 - x_4 - x_5\end{aligned}$$

Cycling

- Is it possible to get the same slack form more than once? Then SIMPLEX has a problem.
- It is in fact possible. To show this we use a specific rules for selecting entering and leaving variables at each iteration.
- The entering variable will always be a nonbasic variable that has the largest positive coefficient in the z-row.
- If two or more basic variables compete for leaving the basis, then the candidate with the smallest subscript will be made to leave.

Cycling - Example

$$\begin{aligned}z &= 0 + 10x_1 - 57x_2 - 9x_3 - 24x_4 \\x_5 &= 0 - 0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4 \\x_6 &= 0 - 0.5x_1 + 1.5x_2 + 0.5x_3 - x_4 \\x_7 &= 1 - x_1\end{aligned}$$

$$\begin{aligned}z &= 0 + 53x_2 + 41x_3 - 204x_4 - 20x_5 \\x_1 &= 0 + 11x_2 + 5x_3 - 18x_4 - 2x_5 \\x_6 &= 0 - 4x_2 - 2x_3 + 8x_4 + x_5 \\x_7 &= 1 - 11x_2 - 5x_3 + 18x_4 + 2x_5\end{aligned}$$

$$\begin{aligned}z &= 0 + 14.5x_3 - 98x_4 - 6.75x_5 - 13.25x_6 \\x_1 &= 0 - 0.5x_3 + 4x_4 + 0.75x_5 - 2.75x_6 \\x_2 &= 0 - 0.5x_3 + 2x_4 + 0.25x_5 - 0.25x_6 \\x_7 &= 1 + 0.5x_3 - 4x_4 - 0.75x_5 - 13.25x_6\end{aligned}$$

Cycling - Example

$$z = 0 + 18x_4 + 15x_5 - 93x_6 - 29x_1$$

$$x_2 = 0 - 2x_4 - 0.5x_5 + 2.5x_6 + x_1$$

$$x_3 = 0 + 8x_4 + 1.5x_5 - 5.5x_6 - 2x_1$$

$$x_7 = 1 - x_1$$

$$z = 0 + 10.5x_5 - 70.5x_6 - 20x_1 - 9x_2$$

$$x_3 = 0 - 0.5x_5 + 4.5x_6 + 2x_1 - 4x_2$$

$$x_4 = 0 - 0.25x_5 + 1.25x_6 + 0.5x_1 - 0.5x_2$$

$$x_7 = 1 - x_1$$

Cycling - Example

$$\begin{aligned}z &= 0 + 24x_6 + 22x_1 - 93x_2 - 21x_3 \\x_4 &= 0 - x_6 - 0.5x_1 + 1.5x_2 + 0.5x_3 \\x_5 &= 0 + 9x_6 + 4x_1 - 8x_2 - 2x_3 \\x_7 &= 1 - x_1\end{aligned}$$

$$\begin{aligned}z &= 0 + 10x_1 - 57x_2 - 9x_3 - 24x_4 \\x_5 &= 0 - 0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4 \\x_6 &= 0 - 0.5x_1 + 1.5x_2 + 0.5x_3 - x_4 \\x_7 &= 1 - x_1\end{aligned}$$

Cycling

- **Claim:** If SIMPLEX fails to terminate then it cycles.
- **Proof:** Suppose that SIMPLEX does not cycle but it fails to terminate. So it must generate infinite number of different slack forms.
- However, the number of different bases is finite. If we can show that a slack form for a given basis is unique then we have a contradiction.

$$z = v + \sum_{j \notin B} c_j x_j$$

$$x_i = b_i - \sum_{j \notin B} a_{ij} x_j \quad \text{for } i \in B$$

$$z = v^* + \sum_{j \notin B} c_j^* x_j$$

$$x_i = b_i^* - \sum_{j \notin B} a_{ij}^* x_j \quad \text{for } i \in B$$

- Let $k \notin B$ and let t be any number. Set $x_k = t$ and set all other nonbasic variables to 0.
- Then $x_i = b_i - a_{ik} t$ for all $i \in B$, $z = v + c_k t$.
- This is a solution to the "LEFT" system. Since "LEFT" and "RIGHT" are equivalent, it is also a solution to the "RIGHT" system. Hence,
 - $b_i - a_{ik} t = b_i^* - a_{ik}^* t$ for all $i \in B$
 - $v + c_k t = v^* + c_k^* t$
- These equalities must hold for all choices of t . Therefore $b_i = b_i^*$, $a_{ik} = a_{ik}^*$ for all $i \in B$, and $v = v^*$, $c_k = c_k^*$.
- Since k was chosen arbitrarily, the two slack forms are identical.

Avoiding Cycling

- Perturb input slightly so that it is impossible to have two basic solutions with the same objective value.
- Always choose the entering and leaving variables with the smallest indices.

Perturbation Method

$$1 \gg e_1 \gg e_2 \gg \dots \gg e_m - 1 \gg e_m > 0$$

$$\begin{aligned} z &= 0 & + & 10x_1 & - & 57x_2 & - & 9x_3 & - & 24x_4 \\ x_5 &= e_1 & - & 0.5x_1 & + & 5.5x_2 & + & 2.5x_3 & - & 9x_4 \\ x_6 &= e_2 & - & 0.5x_1 & + & 1.5x_2 & + & 0.5x_3 & - & x_4 \\ x_7 &= 1 + e_3 & - & x_1 & & & & & & \end{aligned}$$

$$\begin{aligned} z &= 20e_2 & - & 27x_2 & + & x_3 & - & 44x_4 & - & 20x_6 \\ x_5 &= e_1 - e_2 & + & 4x_2 & + & 2x_3 & - & 8x_4 & + & x_6 \\ x_1 &= 2e_2 & + & 3x_2 & + & x_3 & - & 2x_4 & - & 2x_6 \\ x_7 &= 1 - 2e_2 + e_3 & - & 3x_2 & - & x_3 & - & 44x_4 & - & 20x_6 \end{aligned}$$

$$\begin{aligned} z &= 1 + 18e_2 + e_3 & - & 30x_2 & - & 42x_4 & - & 18x_6 & - & x_7 \\ x_5 &= 2 + e_1 - 5e_2 + 2e_3 & - & 2x_2 & - & 4x_4 & + & 5x_6 & - & 2x_7 \\ x_1 &= 1 + e_3 & & & & & & & & - & x_7 \\ x_3 &= 1 - 2e_2 + e_3 & - & 3x_2 & + & 2x_4 & + & 2x_6 & - & x_7 \end{aligned}$$

Small Indices

$$\begin{aligned}z &= 0 + 10x_1 - 57x_2 - 9x_3 - 24x_4 \\x_5 &= 0 - 0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4 \\x_6 &= 0 - 0.5x_1 + 1.5x_2 + 0.5x_3 - x_4 \\x_7 &= 1 - x_1\end{aligned}$$

$$\begin{aligned}z &= 0 + 53x_2 + 41x_3 - 204x_4 - 20x_5 \\x_1 &= 0 + 11x_2 + 5x_3 - 18x_4 - 2x_5 \\x_6 &= 0 - 4x_2 - 2x_3 + 8x_4 + x_5 \\x_7 &= 1 - 11x_2 - 5x_3 + 18x_4 + 2x_5\end{aligned}$$

$$\begin{aligned}z &= 0 + 14.5x_3 - 98x_4 - 6.75x_5 - 13.25x_6 \\x_1 &= 0 - 0.5x_3 + 4x_4 + 0.75x_5 - 2.75x_6 \\x_2 &= 0 - 0.5x_3 + 2x_4 + 0.25x_5 - 0.25x_6 \\x_7 &= 1 + 0.5x_3 - 4x_4 - 0.75x_5 - 13.25x_6\end{aligned}$$

Small Indices

$$z = 0 + 18x_4 + 15x_5 - 93x_6 - 29x_1$$

$$x_2 = 0 - 2x_4 - 0.5x_5 + 2.5x_6 + x_1$$

$$x_3 = 0 + 8x_4 + 1.5x_5 - 5.5x_6 - 2x_1$$

$$x_7 = 1 - x_1$$

$$z = 0 + 10.5x_5 - 70.5x_6 - 20x_1 - 9x_2$$

$$x_3 = 0 - 0.5x_5 + 4.5x_6 + 2x_1 - 4x_2$$

$$x_4 = 0 - 0.25x_5 + 1.25x_6 + 0.5x_1 - 0.5x_2$$

$$x_7 = 1 - x_1$$

Small Indices

$$z = 0 + 24x_6 + 22x_1 - 93x_2 - 21x_3$$

$$x_5 = 0 + 9x_6 + 4x_1 - 8x_2 - 2x_3$$

$$x_4 = 0 - x_6 - 0.5x_1 + 1.5x_2 + 0.5x_3$$

$$x_7 = 1 - x_1$$

$$z = 0 - 20x_6 - 27x_2 + x_3 - 44x_4$$

$$x_5 = 0 + x_6 + 4x_2 + 2x_3 - 8x_4$$

$$x_1 = 0 - 2x_6 + 3x_2 + x_3 - 2x_4$$

$$x_7 = 1 + 2x_6 - 3x_2 - x_3 + 2x_4$$

$$z = 1 - 18x_6 - 30x_2 - 42x_4 - x_7$$

$$x_5 = 2 + 5x_6 - 2x_2 - 4x_4 - 2x_7$$

$$x_1 = 1 - x_7$$

$$x_3 = 1 + 2x_6 - 3x_2 + 2x_4 - x_7$$

Infeasible First Basic Solution

$$\begin{array}{ll} \max & 2x_1 - x_2 \\ \text{s.t.} & 2x_1 - x_2 \leq 2 \\ & x_1 - 5x_2 \leq -4 \end{array}$$

$$\begin{array}{llllll} z & = & 0 & + & 2x_1 & - & x_2 \\ x_3 & = & 2 & - & 2x_1 & + & x_2 \\ x_4 & = & -4 & - & x_1 & + & 5x_2 \end{array}$$

- x_1 and x_2 would be set to 0 in the first basic solution.
- This solution is infeasible.

Auxiliary LP

- We will define a related auxiliary LP.
- This auxiliary LP is feasible and bounded.
- Optimal solution to this auxiliary LP will indicate if the LP is feasible.
- If LP is feasible, then the slack form of this auxiliary LP will yield a feasible basic solution to the LP (and the corresponding slack form).

Auxiliary Linear Program

- L: LP in standard form:

$$\begin{aligned} \max \quad & \sum_{j=1}^n c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j \leq b_i \quad \text{for } i=1,2,\dots,m \\ & x_j \geq 0 \quad \text{for } j=1,2,\dots,n \end{aligned}$$

- L_{aux} : Auxiliary LP:

$$\begin{aligned} \max \quad & -x_0 \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j - x_0 \leq b_i \quad \text{for } i=1,2,\dots,m \\ & x_j \geq 0 \quad \text{for } j=0,1,2,\dots,n \end{aligned}$$

- L_{aux} is bounded and feasible.

INITIALIZE_SIMPLEX

- Let l be the index of the minimum b_i .
- If $b_l \geq 0$, let $N=\{1,2,\dots,n\}$, $B=\{n+1,n+2,\dots,n+m\}$ and return $(N,B,A,b,c,0)$.
- Otherwise form L_{aux} in its slack form.
- $(N,B,A,b,c,v) \leftarrow \text{PIVOT}(N,B,A,b,c,v,l,0)$. This basic solution is feasible for L_{aux} .
- Keep pivoting until SIMPLEX terminates. If the returned objective value is 0, then use the returned slack form (with x_0 removed) and restored objective function as a starting point for SIMPLEX. If the returned objective value is negative, then the original LP is infeasible.

INITIALIZE_SIMPLEX - Example

$$\begin{array}{ll} L: & \max \quad 2x_1 - x_2 \\ & s.t. \quad 2x_1 - x_2 \leq 2 \\ & \quad \quad x_1 - 5x_2 \leq -4 \end{array}$$

$$\begin{array}{ll} L_{\text{aux}} & \max \quad -x_0 \\ & s.t. \quad 2x_1 - x_2 - x_0 \leq 2 \\ & \quad \quad x_1 - 5x_2 - x_0 \leq -4 \end{array}$$

- If the value of the optimal solution to L_{aux} is 0, then if we disregard x_0 , we get a feasible solution to L . It is basic (at most 2 variables are positive, others are 0).
- If the value of the optimal solution to L_{aux} is negative, then at least one of the constraints of L is never satisfied. L is infeasible.

INITIALIZE_SIMPLEX - Example

$$\begin{aligned}
 z & & & & & & - & x_0 \\
 x_3 & = & 2 & - & 2x_1 & + & x_2 & + & x_0 \\
 x_4 & = & -4 & - & x_1 & + & 5x_2 & + & x_0
 \end{aligned}$$

$$\begin{aligned}
 z & & & & & & - & (x_4 + 4 + x_1 - 5x_2) \\
 x_3 & = & 2 & - & 2x_1 & + & x_2 & + & (x_4 + 4 + x_1 - 5x_2) \\
 x_0 & = & 4 & + & x_1 & - & 5x_2 & + & x_4
 \end{aligned}$$

$$\begin{aligned}
 z & = & -4 & - & x_1 & + & 5x_2 & - & x_4 \\
 x_3 & = & 6 & - & x_1 & - & 4x_2 & + & x_4 \\
 x_0 & = & 4 & + & x_1 & - & 5x_2 & + & x_4
 \end{aligned}$$

feasible!!

INITIALIZE_SIMPLEX - Example

$$\begin{aligned}z &= -4 - x_1 + 5x_2 - x_4 \\x_3 &= 6 - x_1 - 4x_2 + x_4 \\x_0 &= 4 + x_1 - 5x_2 + x_4\end{aligned}$$

$$\begin{aligned}z &= -4 - x_1 + 5(4/5 + x_1/5 + x_4/5 - x_0/5) - x_4 \\x_3 &= 6 - x_1 - 4(4/5 + x_1/5 + x_4/5 - x_0/5) + x_4 \\x_2 &= 4/5 + x_1/5 - x_0/5 + x_4/5\end{aligned}$$

INITIALIZE_SIMPLEX - Example

$$\begin{aligned}
 z &= 0 && - && x_0 \\
 x_3 &= 14/5 && - && 9x_1/5 && + && 4x_0/5 && + && x_4/5 \\
 x_2 &= 4/5 && + && x_1/5 && - && x_0/5 && + && x_4/5
 \end{aligned}$$

$$\begin{aligned}
 z &= 0 && + && 2x_1 && - && (4/5 + x_1/5 - x_0/5 + x_4/5) \\
 x_3 &= 14/5 && - && 9x_1/5 && + && 4x_0/5 && && + && x_4/5 \\
 x_2 &= 4/5 && + && x_1/5 && - && x_0/5 && && + && x_4/5
 \end{aligned}$$

$$\begin{aligned}
 z &= -4/5 && + && 9x_1/4 && - && x_4/5 \\
 x_3 &= 14/5 && - && 9x_1/5 && + && x_4/5 \\
 x_2 &= 4/5 && + && x_1/5 && + && x_4/5
 \end{aligned}$$

L is feasible \Leftrightarrow Optimal objective
value of L_{aux} is 0

\Rightarrow Let (s_1, s_2, \dots, s_n) be a feasible solution for L.

Let $s_0=0$. Then $(s_0, s_1, s_2, \dots, s_n)$ is a feasible solution for L_{aux} with the objective value 0.

Since $x_0 \geq 0$ in L_{aux} and the objective is to maximize $-x_0$ (or minimize x_0), $(s_0, s_1, s_2, \dots, s_n)$ is an optimal solution for L_{aux} with the objective value 0.

\Leftarrow Let $(s_0, s_1, s_2, \dots, s_n)$ be the optimal solution for L_{aux} with the objective value 0.

Then $s_0=0$ and (s_1, s_2, \dots, s_n) is a feasible solution for L.

L is infeasible iff L_{aux} has a negative optimal solution

\Rightarrow Suppose that L is infeasible.

– Optimal objective value of L_{aux} is not 0.

– L_{aux} is bounded: Let

$$s_0 = \left| \min_{i=1}^m \{ b_i \} \right|$$

– $(s_0, 0, 0, \dots, 0)$ is a feasible solution for L_{aux} with negative objective value.

\Leftarrow At least one constraint of L_{aux} is binding

$$\sum_{j=1}^n a_{ij} x_j - x_0 = b_i$$

– $-x_0$ is negative implying that

$$\sum_{j=1}^n a_{ij} x_j - x_0 = b_i$$

L_{aux} returns a basic solution for L

- If all b -values in L are nonnegative, then the first slack form of L gives a feasible basic solution and we do not need L_{aux} .
- Assume that $b_i < 0$ is the smallest b -value in L . We have to show that after pivoting all b -values are nonnegative.
- The rest is algebraic manipulation, see page 815-816.

SIMPLEX – Open Issues

- How to decide that LP is feasible? **SOLVED**
- What to do if the initial basic solution is infeasible? **SOLVED**
- How to select entering and leaving variables? **SOLVED**
- How to decide that LP is unbounded? **SOLVED**
- Does SIMPLEX terminate? **SOLVED**
- Does it terminate with an optimal solution?