

## DESIGN PROBLEMS IN ROBUST OPTICAL NETWORKS

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### Abstract

The goal of the paper is two-fold. First, the paper formulates a set of optimization problems relevant for the design of optical WDM networks robust to failures, encompassing demand routing, wavelength assignment and link dimensioning. Two basic protection mechanisms are considered: path diversity and single backup path restoration. The design problems, taking directly into account a scenario of assumed failure situations, are formulated as mixed linear integer programming tasks. For small networks these tasks can be solved directly with the branch and bound approach available e.g. in the CPLEX optimization package. As the considered problems are *NP*-complete, for networks of realistic size heuristic methods are called for. Accordingly, the second goal of the paper is to demonstrate how to apply two proposed stochastic heuristic approaches, namely Simulated Annealing and Simulated Allocation, to the specified problems. It is shown using large network configurations that the latter approach, although not commonly known, turns out to be superior to the former, and yields good sub-optimal solutions in reasonable time. Our numerical results also show what extra spare capacity volume is required by the considered protection mechanisms. This can help solving the tradeoff between the reconfiguration complexity and the extra link and node capacity cost.

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## 1. Introduction

All over the world transport providers are seeking to deploy networks based on the newest optical communication technologies (as WDM). Several all-optical transport network field-trials have been carried out [1,2] whereby alternative network designs could be verified. One important factor enabling economical designs of such networks are methods for determining effective demand routing and resulting deployment of fibers between nodes, taking into account the wavelength clashes, the limited number of wavelengths, and the use of protection mechanisms assuring the robustness requirements.

Several papers have treated the routing and wavelength assignment problems [3,4,5,6]. In [7] the multi-fiber case is taken into account and different protection schemes are compared using integer programming formulations and heuristics. In this paper we address certain extensions of the problems from the above-listed references. In particular we consider the simultaneous design of demands routing, wavelengths assignment and link capacities for optical networks with and without protection. We present new integer programming formulations of the resulting design tasks, corresponding to two different protection mechanisms (path diversity and single backup paths), and discuss new (Simulated Allocation) and known (Simulated Annealing) heuristic methods, applicable for large networks. The design tasks take directly into account scenarios of assumed failure situations, and the constant and capacity-dependent costs of optical links. The effectiveness of the methods is compared with an exact branch and bound approach, using real network configurations.

## 2. Design tasks

In this section we specify several Integer Programming (IP) robust design tasks corresponding to the path diversity protection (PDP) and to single backup-path protection (SBP) mechanisms.

### PDP: Path Diversity Protection

#### indices:

$d=1,2,\dots,D$	demands (between pairs of nodes)
$j=1,2,\dots,m(d)$	paths for flows realizing demand $d$
$c=1,2,\dots,C$	colors (wavelengths) available on the fibers
$e=1,2,\dots,E$	links
$s=0,1,\dots,S$	failure situations ( $s=0$ is the nominal state with no failures)

#### constants:

$h_{ds}$	volume of demand $d$ to be realized in situation $s$ , expressed as the number of light-paths
$a_{edj}$	link-path incidence coefficient: $a_{edj}=1$ if link $e$ belongs to path $j$ realizing demand $d$ , $a_{edj}=0$ otherwise
$\bullet_{es}$	binary link failure coefficient: $\bullet_{es}=0$ if link $e$ is failed in situation $s$ , $\bullet_{es}=1$ if it works; we assume that $\bullet_{e0}=1$
$\bullet_{djs}=\prod_{e: a_{edj}=1} \bullet_{es}$	path failure coefficient: $\bullet_{djs}=0$ if path $j$ of demand $d$ is failed in situation $s$ , $\bullet_{djs}=1$ otherwise
$c_e$	marginal cost of link $e$

#### variables:

$\bullet_{djc}$	flow (number of light-paths) realizing demand $d$ in color $c$ on path $j$ (non-negative integer)
$n_{ce}$	number of times the color $c$ is used on link $e$ (non-negative integer, auxiliary)
$y_e$	capacity of link $e$ expressed in the number of fibers (non-negative integer)

**objective:** minimize  $C(y) = \sum_e c_e y_e$  (1)

#### constraints:

$$\sum_j \bullet_{djs} \sum_c \bullet_{djc} \exists h_{ds} \quad d=1,2,\dots,D, \quad s=0,1,\dots,S \quad (2)$$

$$\sum_d \sum_j a_{edj} \bullet_{djc} = n_{ce} \quad c=1,2,\dots,C, \quad e=1,2,\dots,E \quad (3)$$

$$y_e \exists n_{ce} \quad c=1,2,\dots,C, \quad e=1,2,\dots,E. \quad (4)$$

**SBP: Single Backup-path Protection**
**additional indices:**

- $k=1,2,\dots,l(d,j)$  backup paths for protecting nominal flow realizing demand  $d$  on path  $j$
- $g=1,2,\dots,C$  backup colors

**additional constants:**

- $b_{edjk}$  link-path incidence coefficient:  $b_{edjk}=1$  if link  $e$  belongs to backup path  $k$  protecting path  $j$  of demand  $d$ ,  $b_{edjk}=0$  otherwise
- $\bullet_{djs} = \prod_{e: b_{edjk}=1} \bullet_{es}$  path failure coefficient:  $\bullet_{djs}=0$  if backup path  $k$  protecting path  $j$  of demand  $d$  is failed in situation  $s$ ,  $\bullet_{djs}=1$  otherwise; we require that for each  $s, d, j$  and  $k$ ,  $\bullet_{djs}=0$  implies  $\bullet_{djs}=1$ , i.e. that the paths  $j$  and  $k$  are a failure disjoint@

**variables:**

- $\bullet_{djc}$  nominal flow realizing  $d$  in color  $c$  on path  $j$  (non-negative integer)
- $\bullet_{djkcg}$  backup flow on path  $k$  in color  $g$  protecting nominal flow  $\bullet_{djc}$  on path  $j$  (non-negative integer)
- $\bullet_{djkcg}$  protection flow allocation variable (binary)
- $n_{ces}$  number of times the color  $c$  is used on link  $e$  in situation  $s$  (non-negative integer, auxiliary)
- $y_e$  integer capacity of link  $e$  expressed in the number of fibers (non-negative integer)

**objective:** minimize (1)

**constraints:**

$$\sum_j \sum_c \bullet_{djc} = h_d \quad d=1,2,\dots,D \quad (\text{full restoration is assumed: } h_{ds}=h_d \text{ for } s=0,1,\dots,S) \quad (5)$$

$$\sum_k \sum_g \bullet_{djkcg} = 1 \quad d=1,2,\dots,D, \quad j=1,2,\dots,m(d), \quad c=1,2,\dots,C \quad (6)$$

$$\bullet_{djkcg} \# \bullet_{djkcg} h_d \quad d=1,2,\dots,D, \quad j=1,2,\dots,m(d), \quad k=1,2,\dots,l(d,j), \quad c=1,2,\dots,C \quad (7)$$

$$\sum_k \sum_g \bullet_{djkcg} = \bullet_{djc} \quad d=1,2,\dots,D, \quad j=1,2,\dots,m(d), \quad c=1,2,\dots,C \quad (8)$$

$$\sum_d \sum_j (\bullet_{djs} a_{edj} \bullet_{djc} + (1 - \bullet_{djs}) \sum_k \sum_g b_{edjk} \bullet_{djkcg}) = n_{ces} \quad e=1,2,\dots,E, \quad c=1,2,\dots,C, \quad s=0,1,\dots,S \quad (9)$$

$$y_e \exists n_{ces} \quad c=1,2,\dots,C, \quad e=1,2,\dots,E, \quad s=0,1,\dots,S. \quad (10)$$

The rest of the design tasks considered in Section 3 are obtained from PDP or SDP as follows.

**ND: Nominal Design.** ND is identical with PDP with  $S=0$  (only nominal state considered).

**SBP/FC: SBP with Fixed Colors.** One additional constraint is added to SBP:

$$\sum_{g \neq c} \bullet_{djkcg} = 0 \quad d=1,2,\dots,D, \quad j=1,2,\dots,m(d), \quad k=1,2,\dots,l(d,j), \quad c=1,2,\dots,C. \quad (11)$$

**SBP/NC: SBP with Non-reusable Nominal Capacity.** Constraint (12) substitutes (9):

$$\sum_d \sum_j (a_{edj} \bullet_{djc} + (1 - \bullet_{djs}) \sum_k \sum_g b_{edjk} \bullet_{djkcg}) = n_{ces} \quad e=1,2,\dots,E, \quad c=1,2,\dots,C, \quad s=0,1,\dots,S. \quad (12)$$

**SBP/FC/NC: SBP with Fixed Colors and Non-reusable Nominal Capacity.** Constraint (11) is added to SBP and constraint (12) substitutes (9).

Using binary variables we are able to consider, maintaining the IP formulation, the extended objective function (1) taking into account the constant ‘‘link-opening’’ costs implied by the ducts:

$$C(y) = \sum_e F_e(y_e), \quad F_e(y_e) = c_e y_e + f_e \text{ for } y_e > 0, \text{ and } F_e(y_e) = 0 \text{ for } y_e = 0. \quad (13)$$

To achieve this, for each link  $e$  we introduce an additional binary variable  $\bullet_e$  and add the following (where  $K$  is a large number):

$$\text{objective:} \quad \text{minimize } C(y, \bullet) = \sum_e (c_e y_e + f_e \bullet_e) \quad (14)$$

$$\text{additional constraints:} \quad y_e \leq \bullet_e K, \quad e=1,2,\dots,E. \quad (15)$$

### 3. Design methods

In this section we describe three optimization methods which are applicable to the problems specified in Section 2. As the exact approaches are effective only for small networks, two stochastic heuristic methods, applicable also to large networks, are discussed.

### 3.1. Exact Methods

The tasks defined in Section 2 are linear Integer Programming tasks and hence we have tried an IP solver for them. Such a solver, based on the branch and bound approach, is available e.g. in CPLEX [8]. The CPLEX results (including the results for the corresponding relaxed LP problems) for a small network are presented in Section 3.3.

### 3.2. Heuristic Methods

The exact IP methods are useful mainly because they can yield optimal reference solutions for small networks. For networks of realistic size, however, heuristic methods are called for. We will describe the use of two of these: Simulated Allocation (SAL) and Simulated Annealing (SAN). In both cases the optimization problems are solved through finding suboptimal full allocation states, by allocating and disconnecting the light-paths. Network paths considered by the algorithms are predefined; they can be calculated e.g. with the well known techniques for finding a set of k-shortest paths [9] or a shortest set of k-disjoint paths [10], depending on the problem. For the nominal design case (ND) an allocation state is represented by the following flow vector:

$$x = (x_{djc}: d=1,2,\dots,D, j=1,2,\dots,m(d), c=1,2,\dots,C) \quad (16)$$

with the entries corresponding to flows  $\bullet_{djc}$  in PDP. Entry  $x_{djc}=x_{121}$  specifies that  $x_{121}$  light-paths are used for demand  $d=1$  on its path  $j=2$  using the laser beam with color  $c=1$ , and  $x_{djc}=x_{152}$  implies that  $x_{152}$  light-paths are used for demand  $d=1$  on its path  $j=5$  using the laser beam with  $c=2$ . In the robust design cases the allocation state is represented by the flow vector specifying the amount of light-paths in color  $c$  allocated to nominal path  $j$  of demand  $d$ , protected in color  $g$  on backup path  $k$  (cf. indices in SBP):

$$x = (x_{djkeg}: d=1,2,\dots,D, j=1,2,\dots,m(d), k=1,2,\dots,l(d,j), c=1,2,\dots,C, g=1,2,\dots,C). \quad (17)$$

Flow vector  $x$  determines the necessary link capacities  $y_e(x)$  and the cost  $C(x) = \sum_e F_e(y_e(x))$ .

**Simulated Allocation (SAL).** Simulated Allocation is a meta-heuristic which has been successfully applied to various design tasks (cf.[11,12]). Its pseudo-code is as follows.

---

```

begin
  step:= 0; min_cost:= 4; x:= 0;
  repeat
    step:= step+1;
    if random < q(x) then allocate(x) else if C(x) < min_cost then disconnect1(x) else disconnect2(x);
    if x is a maximal allocation state and C(x) < min_cost then
      begin min_cost:= C(x); x_best:= x end
  until step = step_limit or min_cost = cost_lower_bound
end

```

---

SAL works with partial allocation states. The algorithm starts with the all zero-flow solution, and in each step it chooses (in a probabilistic way) between *allocate*( $x$ ), i.e. adding one or more light-paths to the flow vector  $x$ , or *disconnect*( $x$ ), i.e. removing one or more light-paths from the current solution  $x$ . We require that  $q(x) > 1/2$ , and seek for improving the best so far reached solution whenever a complete allocation state is found. Procedure *allocate*( $x$ ) allocates a bulk of  $M$  light-paths of the same type (it increments one selected entry  $x_{djkeg}$  with the constant  $M$  - the bulk size). The demand  $d$  for allocation is chosen with a probability proportional to the current number of not allocated light-paths; for a given  $d$ , the paths  $(j,k)$  and colors  $(c,g)$  are selected to minimize

the increase of the cost function. Procedure  $disconnect(x)$  is used in two variants:

$disconnect1(x)$ : remove from  $x$  a bulk of  $M$  previously allocated light-paths (for a single  $d$ )

$disconnect2(x)$ : remove from  $x$  all the light-paths which use a randomly chosen set of links.

The second variant is applied if (and only if)  $C(x)$  is greater than  $min\_cost$ .

**Simulated Annealing (SAN).** In our SAN (cf.[13]) approach to network design the solutions  $x$  are full allocation states satisfying constraint (5). We start with a solution generated by  $initialize(x)$  and choose consecutive states ( $neighbor(x)$ ) by swapping a randomly chosen light-path with parameters  $(d,j,c)$  to a light-path with parameters  $(d,jN,cN)$ , with  $jN$  and  $cN$  chosen at random.

---

```

begin
  initialize(x_old); min_cost:= C(x_old); x_best:= x_old; T:= initial_temperature;
  while T  $\exists$  temperature_lower_bound and min_cost > cost_lower_bound
    for counter:= 0 to counter_upper_bound do
      x_new:= neighbor(x_old);  $\bullet$  F:= C(x_new)-C(x_old);
      if  $\bullet$  F  $\neq$  0 then
        begin
          x_old:= x_new; if C(x_new) < min_cost then begin min_cost:= C(x_new); x_best:= x_new end
        end
        else if random < exp{- $\bullet$  F/T} then x_old:= x_new;
      end for;
      T:= TH $\bullet$ 
    end while
  end

```

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### 3.3. Results

We have performed some tests for the six design problems of Section 2 using a small network example with 5 nodes and 7 links. All node pairs are to be connected with 1 to 6 light-paths. Cost function (13) is used, with links' costs  $c_e$  per fiber between 1 and 4, and the constant duct costs  $f_e$  - between 100 and 400 (the duct costs are 100 times greater than the fiber costs).

<b>Table 1.</b>	ND	PDP	SBP	SBP/NC	SBP/FC	SBP/FC/NC
Exact LP	640.28	918.40	885.63	885.63	885.63	885.63
Exact IP	645	1053	991	991	991	991
SAN	869	-	-	1523	-	-
SAL	645	-	-	1016	-	-

The PDP protection scheme is cost inefficient because it does not allow for sharing the protection capacity. The different SBP mechanisms are equivalent in the terms of cost for this small network, but this is by no means a general property. It turns out that the SAL algorithm is significantly better than SAN.

## 4. Numerical results

Below we discuss results of the case studies performed to examine the methods described in Section 3 for the problems specified in Section 2. We have performed tasks ND and SBP/NC for the cost function (13) on an HPJ7000/440MHz for the networks described in Table 2. For the Danish network, two different demand patterns have been used. For SAL and SAN each task was run for 30 min. for the number of wavelengths  $C = 2, 4, 16$ . For each case the stochastic algorithms

have been run 3 times whilst the deterministic IP solver just once.

<i>network</i>	<i>nodes</i>	<i>links</i>	<i>average duct cost (<math>f_e</math>)</i>	<i>average fibre cost (<math>c_e</math>)</i>	<i>demands</i>
Polish	12	18	182	1.82	66
Danish	23	29	214	2.00	21

**Table 2. Network data**

<i>network</i>	<i>volume</i>	<i>C</i>	<i>simulated allocation</i>		<i>simulated annealing</i>		<i>IP</i>	
			<i>average</i>	<i>std.dev.</i>	<i>average</i>	<i>std.dev.</i>	<i>result</i>	<i>status</i>
Polish	1695	2	5677.05	0.00	6373.45	8.45	5298.30	timeout
		4	3950.77	39.00	4881.27	19.26	3607.45	optimal
		16	2568.20	10.48	3812.05	7.47	2324.75	timeout
Danish	63	2	2646.38	0.00	3377.25	172.56	2646.38	optimal
		4	2509.74	2.41	3309.88	135.87	2506.01	optimal
		16	2409.15	4.70	3401.67	211.04	2403.01	timeout
	844	2	6275.49	2.29	6947.73	1.76	6201.16	optimal
		4	4450.85	1.47	6030.99	1098.78	4377.68	optimal
		16	3085.53	3.32	5674.85	2.25	3010.46	optimal

**Table 3. Optimal costs for ND (nominal design) task run for 30 min.**

<i>Network</i>	<i>volume</i>	<i>C</i>	<i>simulated allocation</i>		<i>simulated annealing</i>		<i>IP</i>	
			<i>average</i>	<i>std.dev.</i>	<i>average</i>	<i>std.dev.</i>	<i>result</i>	<i>status</i>
Polish	1695	2	9299.78	51.10	8298.35	23.64	—	timeout
		4	6376.07	19.92	5823.92	11.18	—	timeout
		16	3998.87	5.72	4065.42	3.19	—	memout
Danish	63	2	5128.27	3.65	5444.97	0.33	5139.94	timeout
		4	4894.00	2.53	5221.27	1.50	5253.74	timeout
		16	4721.31	1.50	5070.91	2.61	—	timeout
	844	2	14573.26	16.00	14514.11	3.95	14175.7	timeout
		4	9748.49	7.66	9860.28	5.38	—	timeout
		16	6123.11	3.21	6377.99	1.56	—	memout

**Table 4. Optimal costs for SBP/NC (single backup path, non-reusable nominal capacity) task run for 30 min.**

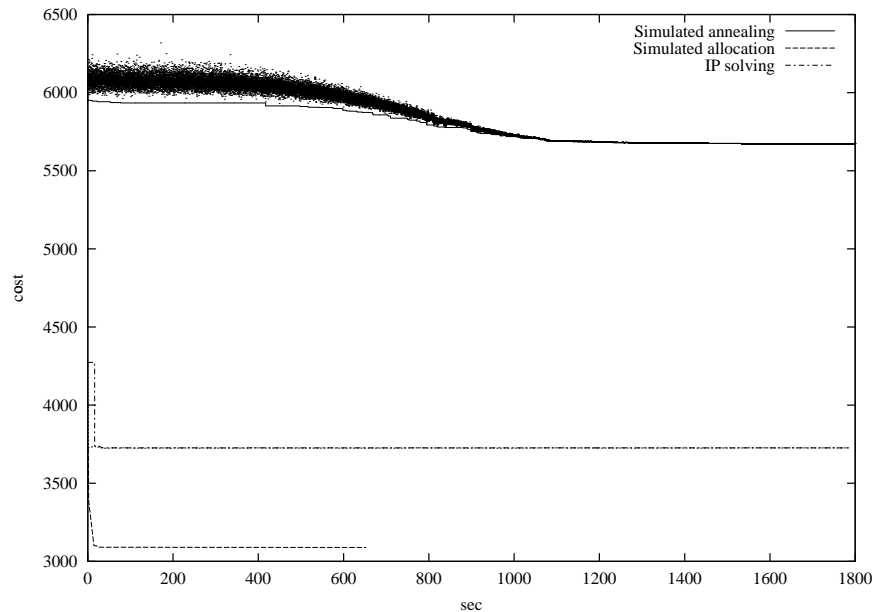
**Figure 1: Optimal cost during optimising ND in the Danish network for 844 lightpaths and 16 wavelengths**

The results are presented in Tab.3 and 4. For SBP/NC, the CPLEX IP solver in many cases does not find a feasible solution before exceeding either the time limit of 30 min. or the memory limit of 1GB. In fact only for task ND with  $C = 2, 4$  in the Danish network, and  $C=2$  in the Polish network does the IP solver finish with an optimal solution within 30 min. Fig.1 shows a typical graph of how the heuristic improvement proceeds; the specs at the top are the consecutive SAN solutions. The three lines show the current best solution cost for the three considered methods. It can be seen that SAL produces a good result very fast, due to the fact that this heuristic has been devised for this kind of tasks. Furthermore, SAL usually produces results at least as good as SAN (in most cases much better). Compared to the IP solver, both heuristics reach a good result fairly fast, which implies that for real-size networks, good heuristics are indispensable.

## 5. Concluding remarks

In the paper we have formulated a set of integer programming design tasks for optical wavelength routed optical networks robust to failures, with the robustness assured by either the path diversity or the single backup path protection mechanisms. The formulations are original and encompass simultaneous design of demands routing, wavelengths assignment and link capacities, with the cost function taking into account the constant duct costs.

For networks of practical size the exact solutions through branch and bound methods using linear programming cannot be achieved in a reasonable time, and hence heuristic approaches must be applied. We have discussed two such approaches: Simulated Annealing (SAN) and a less known method of Simulated Allocation (SAL). It is shown by numerical studies that SAL in most cases outperforms SAN and can be used for finding acceptable suboptimal solutions also for large networks when the exact approaches fail to give even feasible integer solutions. As SAL is also easy to implement (it is similar to the standard simulation of the networks' traffic performance), we propose this method for the use in computer tools aiding the design of optical networks.



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