Exact Evaluation of Blocking in WDM Networks

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Abstract

The all-optical WDM (wavelength division multiplex) networks transmit signals from end to end across intermediate nodes without conversion to and from electrical form. This reduces delay, but imposes a new restriction not seen in electrically switched networks, the *wavelength continuity constraint:* Each link in the network can transmit using a limited number of colours (wavelengths), and when routing a connection, all links along the route must use the same colour.

This added constraint increases the probability of network congestion. We present a novel way of using existing analytical network analysis tools to perform this estimation, changing only the link/route table which is passed to the analyzer.

Assuming Poisson distributed arrival processes, we perform the analysis on some small example networks, and find that the end-to-end call congestion decreases for WDM networks as the number of wavelengths available per link increases. For direct (one-hop) connections, congestion is *lower* in the WDM network than in the electronically switched counterpart, whereas indirect connections (multi-hop) experience higher congestion, because the wavelength constraint is enforced.

As the routing problem with the wavelength continuity constraint is NP-complete, we cannot hope to find efficient algorithms for exact calculation of large networks, and a topic for future research is to find methods for good approximations.

1 Introduction

Based on classical teletraffic theory, several tools for calculating end-to-end blocking characteristics of electronically circuit-switched networks have been constructed. They take as input a *link/route* table, cf. Fig. 1, and produce as output the end-to-end blocking probability for each route.

In this article we consider the problem of calculating the blocking probabilities in circuit-switched optical WDM (wavelength division multiplex) networks without wavelength converters. In an optical WDM network, each link transmits information using a number of different colours; each colour can support just one connection. When a connection uses a path that spans several links, it must use the same colour on every link—this is what is called the *wavelength continuity constraint*. To remove this constraint at a node—that is, to allow a connection to use an outgoing colour that differs from the incoming colour—an optical wavelength converter must be installed. However, these components are not commercially available today, so there is a need for assessing what effect the wavelength continuity constraint has on the blocking probabilities.

To this end, we use the existing tools for electrically circuit-switched networks, by transforming the link/route table into a general constraint/route table which describes the added effect of the wavelength continuity constraints, and then using it as before as input to the tools.

2 Routing traffic demands

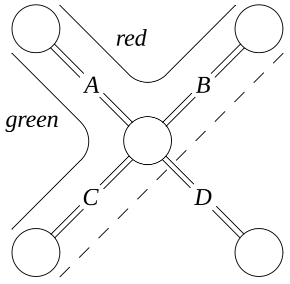
Traffic demands can be classified as *static* or *dynamic.* Static traffic demands are known for all routes before the routing of the demands begin, whereas dynamic traffic demands are routed one at a time, using only the knowledge of the current network state. When dealing with WDM networks, not only must a route be selected, but also a colour which is free on all links of the route must be available. A dynamic traffic demand can be routed and assigned a colour according to various strategies, including random, most-used, least-used, and first-fit allocation [Zhu et al., 1999].

The blocking probabilities computed by the method discussed in this paper assume an optimal colour assignment whenever a traffic demand is routed. That is, if all the demands, including the new one, *can* somehow be assigned colours, the demand is fulfilled. For dynamic traffic demands, this will sometimes require reassignment of the colours on existing routes to free a common colour on all links along a route.

3 Transforming the link/route table for WDM networks

3.1 Routes, links and virtual links

The network we consider consists of a set of links $L = \{l_1, \ldots, l_n\}$, and these are combined to form a set of routes $R = \{r_1, \ldots, r_m\}$, where $r_i \subseteq L$. Each route r_i is also considered to be a *virtual link.* This information is what is present in the link/route table for a classic circuit-switched network; the link/route table for a 4-link network is shown in Fig. 1. This network is *fully routed,* i.e. all possible link combinations, including multicast connections, are available.



		Routes														
		A	*B*	*C*	*D*	*A* *B*	*A* *C*	*A* *D*	*B* *C*	*B* *D*	*C* *D*	*A* *B* *C*	*A* *B* *D*	*A* *C* *D*	*B* *C* *D*	*A* *B* *C* *D*
Links	*A*	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1
	B	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
	C	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1
	D	0	0	0	1	0	0	1	0	1	1	0	1	1	1	1

Figure 1: *Four-link star network and its link/route table.*

Each row of the table can be interpreted as a numerical constraint where the route names are variables indicating the number of connections made along the corresponding route. Assuming link A has a capacity of 2 connections, the first row of the table reads

$$1A + 0B + 0C + 0D + 1AB + 1AC + 1AD + 0BC + 0BD + 0CD + 1ABC + 1ABD + 1ACD + 0BCD + 1ABCD \leq 2$$

If connections are requested for routes AB and AC as shown, this reduces to

$$0 + 0 + 0 + 0 + 1 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \leq 2,$$

confirming that both requests can be fulfilled. When a third request for route BC is included, the top three rows of the table read

$$\begin{aligned} & 1A{+}0B{+}0C{+}0D{+}1AB{+}1AC{+}1AD{+}0BC{+}0BD{+}0CD{+}1ABC{+}1ABD{+}1ACD{+}0BCD{+}1ABCD \\ = {} & 0 + 0 + 0 + 0 + \ 1 \ + \ 1 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \quad \leq 2 \end{aligned}$$

$$\begin{aligned} & 0A{+}1B{+}0C{+}0D{+}1AB{+}0AC{+}0AD{+}1BC{+}1BD{+}0CD{+}1ABC{+}1ABD{+}0ACD{+}1BCD{+}1ABCD \\ = {} & 0 + 0 + 0 + 0 + \ 1 \ + \ 0 \ + \ 0 \ + \ 1 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \quad \leq 2 \end{aligned}$$

$$\begin{aligned} & 0A{+}0B{+}1C{+}0D{+}0AB{+}1AC{+}0AD{+}1BC{+}0BD{+}1CD{+}1ABC{+}0ABD{+}1ACD{+}1BCD{+}1ABCD \\ = {} & 0 + 0 + 0 + 0 + \ 0 \ + \ 1 \ + \ 0 \ + \ 1 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \ + \ 0 \quad \leq 2 \end{aligned}$$

confirming that the request can be fulfilled in an electrically switched network. However, if it is a WDM network with 2 colours on every link, say *red* and *green*, the request cannot be fulfilled, because there is no common free colour on links B and C, as can be seen in Fig. 1. To solve this problem, we extend the table; in this case we must add—among others—a row with '1's in the AB, AC and BC columns.

We will end up with a table where a row does not necessarily corresponds to a physical link, but is a general constraint. When all the constraints are satisfied for a given set of requested routes, the connections can be routed in the WDM network.

3.2 Constructing the constraint/route table

The constraint/route table $\mathcal{C}$ is constructed in two steps:

1. First, a *link exclusion table* is constructed
2. Based on this, the constraint/route table $\mathcal{C}$ is then constructed.

Constructing the link exclusion table. The link exclusion table is an $R \times R$ table. Each column represents a route, and each row represents a virtual link, having a '1' in the corresponding column. Each of remaining columns contains a '0' if the route has no (real) links in common with the virtual link, or otherwise a '?'. For the fully-routed 4-link star network in Fig. 1 we obtain the link exclusions shown in Table 1.

Constructing constraint/route table $\mathcal{C}$. Given two rows of the link exclusion table, $row_x = x_1 x_2 \ldots x_m$ and $row_y = y_1 y_2 \ldots y_m$, we say that row_x *matches* row_y, unless $\{x_i, y_i\} = \{0, 1\}$ for some i. Thus, for instance, given

$$\begin{array}{lcccccccccccccccc} row_C & = & 0 & 0 & 1 & 0 & 0 & ? & 0 & ? & 0 & ? & ? & 0 & ? & ? & ? \\ row_{AB} & = & ? & ? & 0 & 0 & 1 & ? & ? & ? & ? & 0 & ? & ? & ? & ? & ? \\ row_{ABC} & = & ? & ? & ? & 0 & ? & ? & ? & ? & ? & ? & 1 & ? & ? & ? & ? \end{array}$$

		Routes														
		A	*B*	*C*	*D*	*AB*	*AC*	*AD*	*BC*	*BD*	*CD*	*ABC*	*ABD*	*ACD*	*BCD*	*ABCD*
Virtual links	*A*	1	0	0	0	?	?	?	0	0	0	?	?	?	0	?
	B	0	1	0	0	?	0	0	?	?	0	?	?	0	?	?
	C	0	0	1	0	0	?	0	?	0	?	?	0	?	?	?
	D	0	0	0	1	0	0	?	0	?	?	0	?	?	?	?
	AB	?	?	0	0	1	?	?	?	?	0	?	?	?	?	?
	AC	?	0	?	0	?	1	?	?	0	?	?	?	?	?	?
	AD	?	0	0	?	?	?	1	0	?	?	?	?	?	?	?
	BC	0	?	?	0	?	?	0	1	?	?	?	?	?	?	?
	BD	0	?	0	?	?	0	?	?	1	?	?	?	?	?	?
	CD	0	0	?	?	0	?	?	?	?	1	?	?	?	?	?
	ABC	?	?	?	0	?	?	?	?	?	?	1	?	?	?	?
	ABD	?	?	0	?	?	?	?	?	?	?	?	1	?	?	?
	ACD	?	0	?	?	?	?	?	?	?	?	?	?	1	?	?
	BCD	0	?	?	?	?	?	?	?	?	?	?	?	?	1	?
	ABCD	?	?	?	?	?	?	?	?	?	?	?	?	?	?	1

Table 1: *Link exclusion table for the fully routed 4-link network.*

row_C matches row_{ABC} but not row_{AB}, and row_{AB} matches row_{ABC}. We also define an addition operator $+$ by $row_x + row_y = row_z$ where $z_i = x_i + y_i$, $0+0=0$, $1+1=1$, $?+y_i = y_i$ and $x_i + ? = x_i$. Thus, for instance

$$\begin{array}{c ccccccccccccccc} + & ? & 0 & 1 & 0 & 0 & 1 & 0 & ? & 0 & ? & ? & 0 & ? & ? & ? \\ & 0 & 0 & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & 1 & ? \\ \hline = & 0 & 0 & 1 & 0 & 0 & 1 & 0 & ? & 0 & ? & ? & 0 & ? & 1 & ? \end{array}$$

	Routes														
	A	*B*	*C*	*D*	*AB*	*AC*	*AD*	*BC*	*BD*	*CD*	*ABC*	*ABD*	*ACD*	*BCD*	*ABCD*
Constraints	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1
	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1
	0	0	0	1	0	0	1	0	1	1	0	1	1	1	1
	0	0	0	0	1	1	1	0	0	0	1	1	1	1	1
	0	0	0	0	1	1	0	1	0	0	1	1	1	1	1
	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1
	0	0	0	0	1	0	0	1	1	0	1	1	1	1	1
	0	0	0	0	0	1	1	0	0	1	1	1	1	1	1
	0	0	0	0	0	1	0	1	0	1	1	1	1	1	1
	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1
	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

Table 2: *Constraint/route table for the fully routed 4-link WDM network in Fig. 1.*

Given the link exclusion table $[row_1, \ldots, row_m]$, we now construct the constraint/route table

$$\begin{array}{cccccccccccccccccl}
 & 1 & 0 & 0 & 0 & ? & ? & ? & 0 & 0 & 0 & ? & ? & ? & 0 & ? & row_A \\
+ & ? & ? & 0 & 0 & 1 & ? & ? & ? & ? & 0 & ? & ? & ? & ? & ? & row_B \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & ? & ? & 0 & 0 & 0 & ? & ? & ? & 0 & ? & \\
+ & ? & 0 & ? & 0 & ? & 1 & ? & ? & 0 & ? & ? & ? & ? & ? & ? & row_{AB} \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & 1 & ? & 0 & 0 & 0 & ? & ? & ? & 0 & ? & \\
+ & ? & 0 & 0 & ? & ? & ? & 1 & 0 & ? & ? & ? & ? & ? & ? & ? & row_{BC} \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & ? & ? & ? & 0 & ? & \\
+ & ? & ? & ? & 0 & ? & ? & ? & ? & ? & ? & 1 & ? & ? & ? & ? & row_{ABC} \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & ? & ? & 0 & ? & \\
+ & ? & ? & 0 & ? & ? & ? & ? & ? & ? & ? & ? & 1 & ? & ? & ? & row_{ACD} \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & ? & 0 & ? & \\
+ & ? & 0 & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & 1 & ? & ? & row_{BCD} \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & ? & \\
+ & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & 1 & row_{ABCD} \\
\cline{1-16}
= & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 &
\end{array}$$

Figure 2: *Computing the first element of $\mathcal{C}$ for the fully routed 4-link network.*

$\mathcal{C}$ by setting $\mathcal{C} \leftarrow \{\}$, and performing a function call build$(??\ldots?, 1)$, where build is defined by

```
build$(row, i)$ =
  **if** $? \notin row$ **then**
    $\mathcal{C} \leftarrow \mathcal{C} \cup row$
  **else**
    **for** $j \leftarrow i, \ldots, m$ **do if** $row$ matches $row_j$ **then** build$(row + row_j, j + 1)$
```

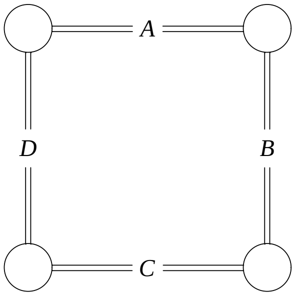
Thus the first element to go into $\mathcal{C}$ for the fully routed 4-link network is computed as shown in Fig. 2, and the final constraint/route table for this WDM network is shown in Table 2.

4 Experiments

The tools which work with link/route tables for computing end-to-end blocking probabilities are often able to handle both an Erlang model where blocked calls are cleared, and a Molina model [Molina, 1922] where blocked calls are held. Call arrivals can be modeled as a state dependent Poisson process, and the models are generally insensitive to the holding time distribution. In the experiment we report on here, we fix the peakedness factor for arrivals at 1, i.e. a state-independent Poisson process.

We calculate the blocking probabilities for two 4-link networks, shown in Figs. 1 and 3. The offered traffic for both networks is shown in Table 3, and is a uniform distribution: given the set of routes using a specific link, the probability of chosing a specific one is independent of its length. The offered load for each link is 1 Erlang.

The results can be seen in Figs. 4 and 5. As expected, congestion in WDM networks decreases with increasing number of wavelengths, due to statistic multiplexing. Furthermore, paths with 2 or 3 hops experience up to 10% higher congestion in WDM networks, compared



	Routes											
	A	*B*	*C*	*D*	*AB*	*AD*	*BC*	*CD*	*ABC*	*ABD*	*ACD*	*BCD*
Constraints	1	0	0	0	1	1	0	0	1	1	1	0
	0	1	0	0	1	0	1	0	1	1	0	1
	0	0	1	0	0	0	1	1	1	0	1	1
	0	0	0	1	0	1	0	1	0	1	1	1
	0	0	0	0	1	1	0	0	1	1	1	1
	0	0	0	0	1	0	1	0	1	1	1	1
	0	0	0	0	0	1	0	1	1	1	1	1
	0	0	0	0	0	0	1	1	1	1	1	1

Figure 3: *Four-link ring network and its constraint/route table.*

	A	*B*	*C*	*D*	*AB*	*AC*	*AD*	*BC*	*BD*	*CD*	*ABC*	*ABD*	*ACD*	*BCD*	*ABCD*
star	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$	$k/8$
ring	$k/6$	$k/6$	$k/6$	$k/6$	$k/6$	–	$k/6$	$k/6$	–	$k/6$	$k/6$	$k/6$	$k/6$	$k/6$	–

Table 3: *Offered load used in the experiments with the star and ring network. The number of connections (wavelengths) per link is denoted by k.*

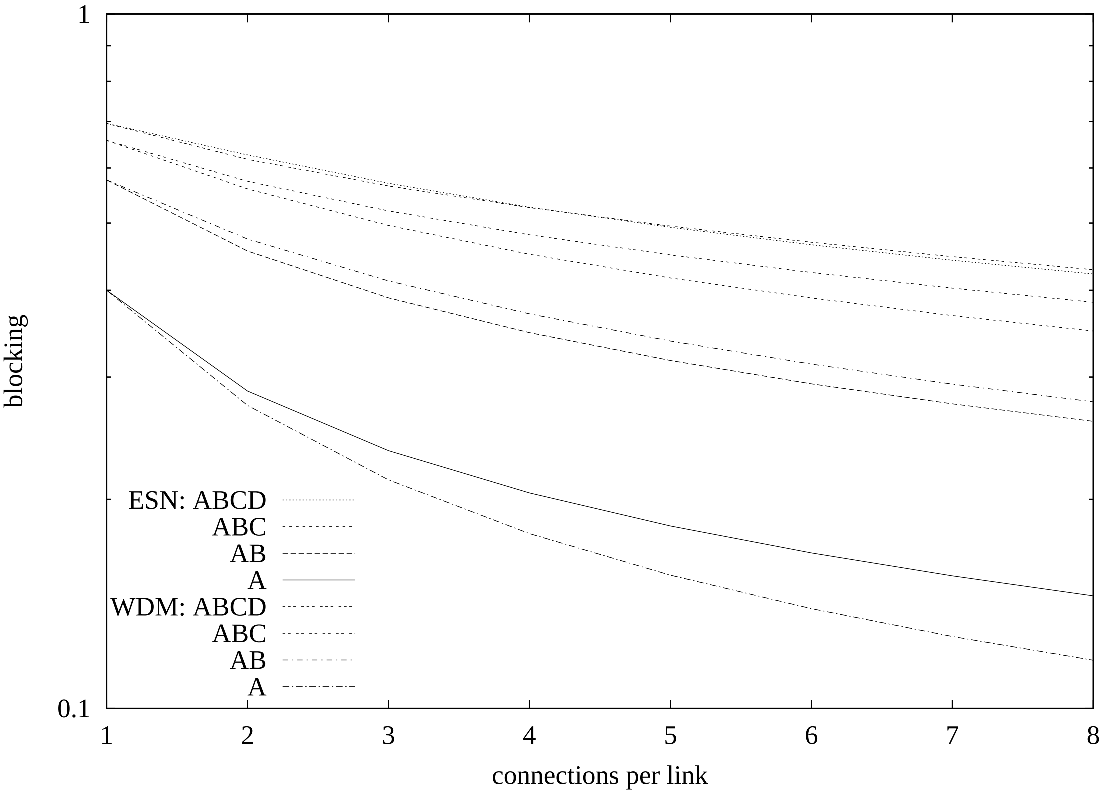


Figure 4: *Blocking probabilities for the 4-link star network.*

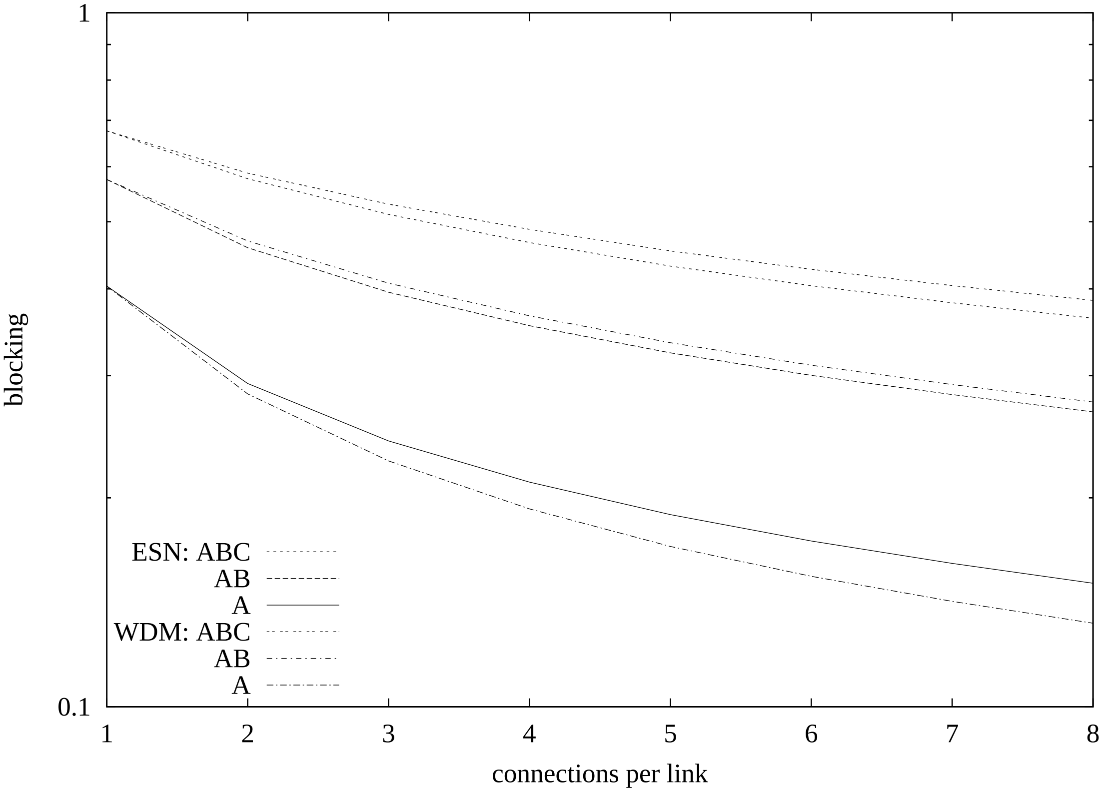


Figure 5: *Blocking probabilities for the 4-link ring network.*

to their electronically switched counterparts, due to the wavelength continuity constraint. This causes the direct (one-hop) routes to receive up to 19% *lower* congestion in WDM networks, cf. Fig. 6. This could indicate that trunk reservation strategies should be reconsidered when dealing with WDM networks without wavelength converters.

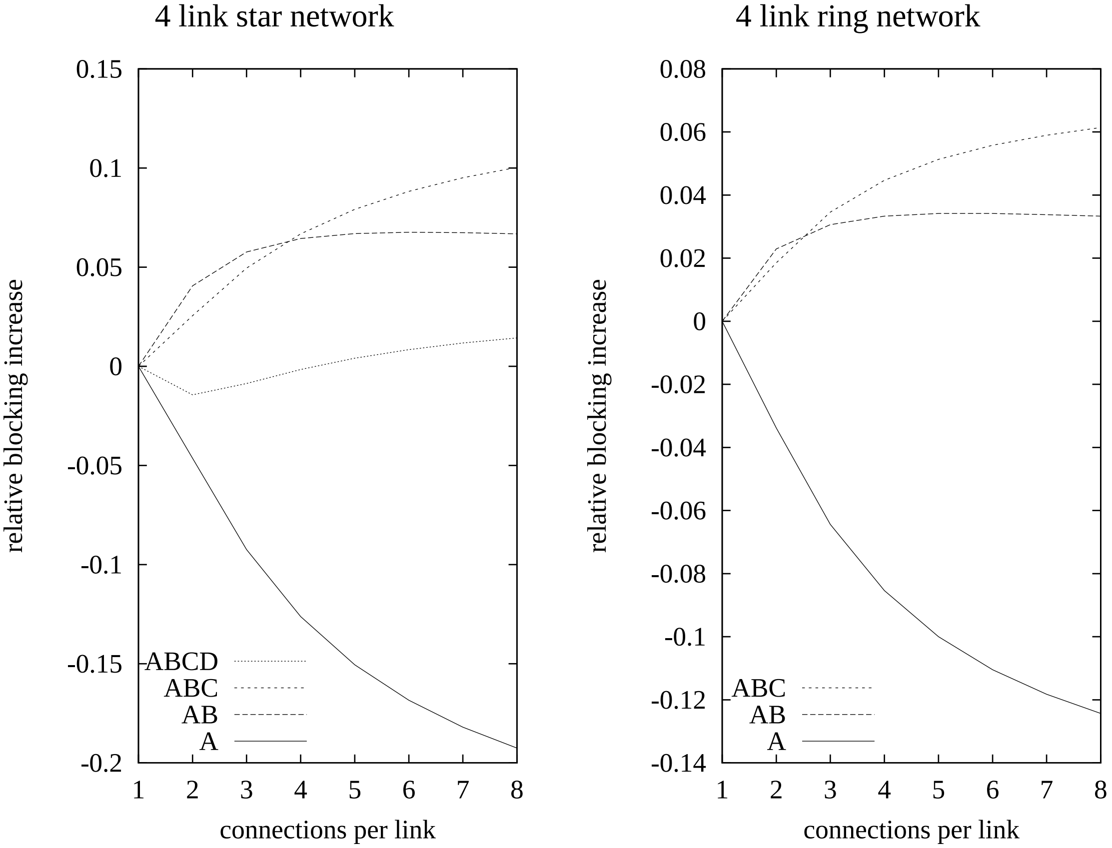


Figure 6: *Relative increase in blocking from electrically switched to WDM networks.*

5 Conclusion

We have shown that it is possible to re-use the existing blocking calculation tools for calculating blocking probabilities in WDM networks. This is done by generalising the link/route tables to constraint/route tables, which include restrictions for enforcing the wavelength continuity constraint. We have shown how this can be achieved using a two-step algorithm: first building a link exclusion table, and then calling a recursive function, to construct the rows of the constraint/route table.

The results from running the blocking calculation programs on constraint/route tables for two small WDM networks show that direct (one-hop) paths experience lower blocking in WDM networks, at the cost of paths with 2 or 3 hops experiencing higher blocking.

Acknowledgements This paper is based on a masters thesis of Brustenga [1999].

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