Signal Processing First

Lab 19: The Fast Fourier Transform

Pre-Lab and Warm-Up: You should read at least the Pre-Lab and Warm-up sections of this lab assignment and go over all exercises in the Pre-Lab section before going to your assigned lab session.

Verification: The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. One of the laboratory instructors must verify the appropriate steps by signing on the **Instructor Verification** line. When you have completed a step that requires verification, simply demonstrate the step to the TA or instructor. Turn in the completed verification sheet to your TA when you leave the lab.

Lab Report: It is only necessary to turn in a report on Section 4 with graphs and explanations. You are asked to **label** the axes of your plots and include a title for every plot. In order to keep track of plots, include your plot *inlined* within your report. If you are unsure about what is expected, ask the TA who will grade your report.

1 Introduction & Objective

The goal of the laboratory project is to introduce the Fast Fourier Transform (FFT) algorithm for efficient computer calculation of the Fourier transform and to investigate some of the Fourier Transform's properties.

2 Background

2.1 The Fast Fourier Transform

Suppose `g` is an array of N values representing the time signal $g[n] = g(nT_s)$. The MATLAB command

```
>> G = fft(g);
```

causes MATLAB to compute the discrete Fourier transform of the time signal $g(nT_s)$ and place the result in array `G`. The array `G` represents a spectrum $G(n\Delta f)$, also of N values. Remember that MATLAB numbers its array elements starting with one. This means that $g[0]$ is stored in array element `g(1)` and $g((N-1)T_s)$ is stored in `g(N)`. Similarly, $G(0)$ is stored in array element `G(1)` and $G((N-1)\Delta f)$ is stored in `G(N)`. For greatest computational efficiency, N should be a power of two. If $g(nT_s), n = 0, ..., N-1$ is a time signal, the Fourier transform that MATLAB calculates is given by

$$G(k\Delta f) = \sum_{k=0}^{N-1} g(nT_s) e^{-j2\pi kn/N}, \quad n = 0, ..., N-1. \tag{1}$$

Note that T_s represents the time between values of $g(kT_s)$. It turns out that the frequency interval Δf between values of $G(n\Delta f)$ is given by

$$\Delta f = \frac{1}{NT_s}$$

Given the array `G`, the MATLAB command

```
>> g = ifft(G);
```

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calculates the inverse Fourier transform given by

$$g(nT_s) = \frac{1}{N} \sum_{k=0}^{N-1} G(k\Delta f) e^{+j2\pi nk/N}, \quad k = 0, ..., N-1. \tag{2}$$

The Fourier transform given by equation 1 is called a discrete Fourier transform or DFT. MATLAB uses the discrete transform because MATLAB cannot store continuous-time signals. MATLAB uses an efficient algorithm called the Fast Fourier Transform (FFT) to calculate the discrete Fourier transform. The discrete Fourier transform has properties that are similar to those of the familiar continuous Fourier transform. There is one important difference. The spectrum $G(k\Delta f)$ defined in equation 1 above is periodic in frequency with period $f_s = N\Delta f$. This periodicity is a consequence of the discrete-time nature of the time signal $g(nT_s)$. One period of the spectrum extends from frequency 0 to frequency $(N-1)\Delta f$. The positive frequency components lie between frequency 0 and frequency $(\frac{N}{2}-1)\Delta f$. The spectral components from frequency $N\Delta f/2$ to $(N-1)\Delta f$ are repeats of the negative frequency components that lie between frequencies $-(\frac{N}{2})\Delta f$ and $-\Delta f$ respectively. Because the spectrum $G(k\Delta f)$ is defined only at discrete values of frequency, the FFT algorithm considers the time function $g(nT_s)$ to be periodic with period NT_s. Consequently, the N-value array g you define will be interpreted as one period of an infinite-duration periodic signal. The spectrum $G(k\Delta f)$ defined in equation (1) is actually the Fourier transform of the periodic signal.

2.2 Plotting the Spectrum

If $G(k\Delta f)$ is plotted against frequency, zero hertz will appear on the left of the graph. The positive frequency components will appear to the right of zero, followed farther to the right by the negative frequency components. Because this is contrary to convention, MATLAB provides a function to rearrange the components of the array `G` to place the negative frequency components to the left of zero. The command

```
>> H = fftshift(G);
```

will create an array `H` that represents a spectrum $H(n\Delta f)$ whose DC component is in the center as expected. Before plotting `H` (or `G`), recall that these arrays may contain complex numbers. The command `plot(H)` will cause MATLAB to plot the imaginary part against the real part. This usually gives an interesting graph, but probably not the one you had in mind! You may obtain the magnitude spectrum by the command `M = abs(H)`, and the angle spectrum by `a = angle(H)`. You may also want to use the commands `real()` and `imag()` to find the real and imaginary parts of the signals you are examining. Tip: if `H` is real, plot it. If `H` is complex, plot `abs(H)` and `angle(H)`.

3 Pre-Lab

Sketch the Fourier Transform for each of the signals given in the procedure. Record them in your lab notebook and bring a photocopy of your notebook to the lecture before lab.

4 Lab Exercises

Because of the requirement that the number of samples be a power of two, we will let all of the time signals in this lab project consist of $N = 512$ samples having a total duration of $500\mu s$. (What does this make T_s? What is Δf?) You can generate a time axis and a frequency axis for your graphs by

```
>> tt = linspace(0,500e-6 - Ts,N);
>> ff = linspace(-(N/2)*deltaf,((N/2)-1)*deltaf,N);
```

Don't forget to do

```
>> G = fftshift(G);
```

so the spectrum matches the values on the axis.

1. A discrete time "unit impulse" is defined by the time signal

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

   Let the time signal be a unit impulse. (Remember that $N = 512$.) Compute and plot the spectrum. Verify that your spectrum is correct by evaluating equation (1) by hand. MATLAB Hint: If you type:

```
set(gcf, 'PaperPosition', [0.5, 0.5, 7.5,1 0])
```

   before printing, your plots will be better spaced on the page.

2. Let the time signal be $g[n] = g(nT_s) = 1$. Compute and plot the spectrum. Verify that your spectrum is correct by substituting the spectrum you obtain into equation (2) and showing by hand that you obtain $g(nT_s)$ back again.

3. Let the time signal be a single pulse extending from $t = -16T_s$ to $t = 16T_s$. (Remember, the time signal is interpreted as periodic!). Compute and plot the spectrum. Verify that it is correct by comparing with the conventional Fourier transform of a continuous-time pulse.

4. Let the time signal be the pulse of Step 3 above, but extending from $t = -32T_s$ to $t = 32T_s$. Compare its spectrum with the spectrum obtained in Step 3.

5. Let the time signal be a cosine of amplitude one whose frequency is chosen so that it has exactly 32 cycles in $500\mu s$. Compute and plot the spectrum. Now verify your result by substituting the spectrum you obtain into equation (2) and showing that you recover the original cosine. (This is much easier than substituting a cosine into equation (1) to verify the spectrum.)

6. Let the time signal be a cosine of amplitude one whose frequency is 65 kHz. Compute and plot the spectrum. Compare the result with the spectrum you obtained in Step 5 above.

7. Let the time signal be an "RF pulse" of frequency 64 kHz and duration $64T_s$.

Instructor Verification (separate page)

5 Report

Include the required spectra from Steps 1 through 7. Verify the correctness of your results as requested. Your report need not contain very much writing, but be sure that what you do write is correct, supports or explains the graphical data, and uses good English. Only one per group needs to do the report.

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Lab 19
INSTRUCTOR VERIFICATION SHEET

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ______________________________ Date of Lab: __________

Signals 1-7

Verified:____________________ Date/Time:__________

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