thanks for your proof of inconsistency:

(1) \( A \times 1 = A \)  
(2) \( A = A \times 1 \)  
(3) \( B \times 1 = B \)  
(4) Assuming \( A \times 1 = B \times 1 \) conditionally for step 4 only
   (4a) \( A \times 1 = B \)  
   (4b) \( A = B \)  
   (4c) \( 1 = 1 \)  
(5) \( A \times 1 = B \times 1 \)  
(6) \( A \times 1 = B \)  
(7) \( A = B \)

As you say, the problem is that the Fix-rule and the isomorphism axiom \( A = A \times 1 \) are inconsistent. This axiom basically breaks the contractiveness assumed in the Fix-rule. I think the best way of seeing this is by writing down the operational interpretation of the proof above (in the sense we’ve given in the paper -- more on that is in Michael’s MS thesis):

\( \text{proj}_A: A \times 1 \rightarrow A \) (projection on first component)  
\( \text{inj}_A = \text{id}_A \times 1 = \lambda: A. (x, ()) : A \rightarrow A \times 1 \) (pairing with unit element)

\( F = \text{fix} f: A \times 1 \rightarrow B \times 1. (\text{proj}_B \circ f \circ \text{inj}_A) \times \text{id}_1 \)

or

\( F(x, ()) = (\text{proj}_B (F(x, ())), ()) \)

where \( \text{proj}_1(A, B): A \times B \rightarrow A \) is the general projection on the first component of pairs.

\( G: A \rightarrow B = \text{proj}_B \circ F \circ \text{inj}_A \)

Now let \( A \) be any type (say the integers) and \( B \) any other type (say the booleans -- we can also take the bottom type, consisting only of nonterminating computations). Then evaluating \( G(5) \) does not terminate, even under lazy evaluation, since \( G(5) \) results in a call to \( F(5, ()) \), which results in a call to \( \text{proj}_B \) (due to the demand of the ‘outside’ \( \text{proj}_B \) in the original call \( G(5) \)), which in turn results in a call to \( F(5, ()) \), and so on.

The way I look at it is that the culprit here is \( \text{proj}_B \), the half of the isomorphism mapping from \( B \times 1 \) to \( B \), since it ‘wipes out’ a constructor (the pair constructor) and is in this sense ‘expansive’ (it basically undoes the contraction step of the outermost pair constructor in the definition of \( F \). So \( F \) is not contractive.
I think it should be possible to characterize (or at least give a useful sufficient condition for) contractiveness in the term interpretation of the coinductive axiomatization so that the Fix rule is only applicable ('truly', semantically) contractively. That is, the applicability of the fix rule is a function of the proof it is supposed to be applied to. Since proofs are not evident in conventional logical rules without a term interpretation, this may not be palatable for esthetic or other reasons for you. (This is also the reason why we combined the Fix rule with the Arrow rule, since the Arrow rule provides a sufficient condition for contractiveness in our setting and thus makes a 'term-less' axiomatization possible.)

I'll get back to you on dates. Best regards,
Fritz