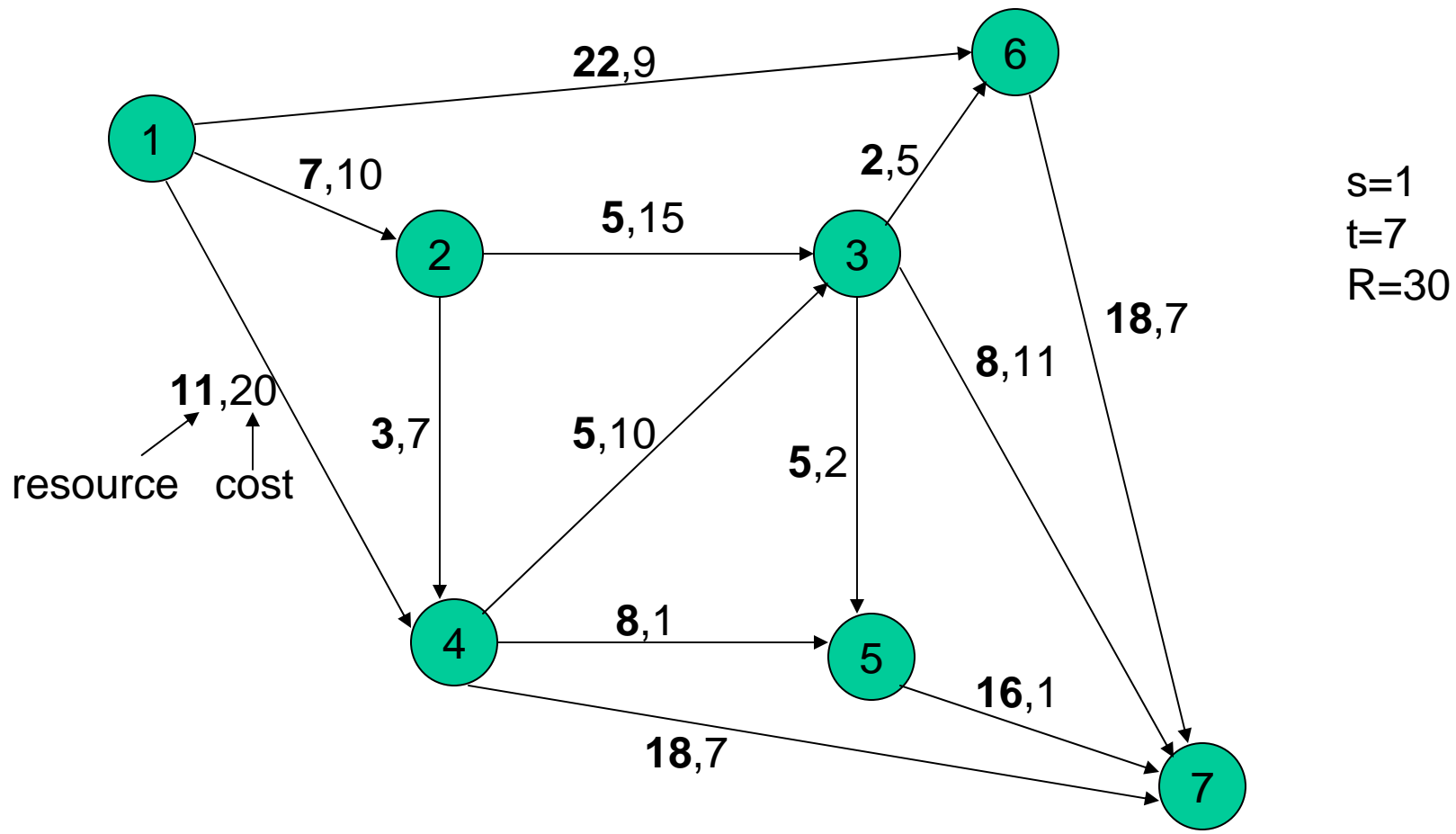


# Lagrangian - Exercise



Min resource path (1,6,7): **40**,16

Min cost path (1,2,3,7): **20**,36

$$f_A(\lambda) = 16 + \lambda(40 - 30) = 16 + 10\lambda$$

$$f_B(\lambda) = 36 + \lambda(20 - 30) = 36 - 10\lambda$$

$$f_A(\lambda) = f_B(\lambda) \Rightarrow \lambda = 1$$

$$f_A(\lambda) = 26$$

Find min  $c + (1)r$  path

(1,2,4,7) of reduced cost 52

$$z(\lambda) = 52 - 30(1) = 22$$

$$z(\lambda) < f_A(\lambda) \Rightarrow \text{repeat}$$

Min red. cost path (1,2,4,7): **28**,24

$$f_A(\lambda) = 16 + \lambda(40 - 30) = 16 + 10\lambda$$

$$f_B(\lambda) = 24 + \lambda(28 - 30) = 24 - 2\lambda$$

$$f_A(\lambda) = f_B(\lambda) \Rightarrow \lambda = 2/3$$

$$f_A(\lambda) = 22 \frac{2}{3}$$

Find min  $c + (2/3)r$  path

(1,2,4,5,7) of reduced cost 41  $\frac{2}{3}$

$$z(\lambda) = 41 \frac{2}{3} - 30(2/3) = 21 \frac{2}{3}$$

$$z(\lambda) < f_A(\lambda) \Rightarrow \text{repeat}$$

Min red. cost path (1,2,4,5,7): **34**,19

$$f_A(\lambda) = 19 + \lambda(\mathbf{34}-30) = 19+4\lambda$$

$$f_B(\lambda) = 24 + \lambda(\mathbf{28}-30) = 24-2\lambda$$

$$f_A(\lambda) = f_B(\lambda) \Rightarrow \lambda = 5/6$$

$$f_A(\lambda) = 22 \frac{1}{3}$$

Find min  $c + (5/6)r$  path

(1,2,4,5,7) of reduced cost  $41 \frac{2}{3}$

$$z(\lambda) = 37 \frac{1}{3} - 30 (5/6) = 22 \frac{1}{3}$$

$$z(\lambda) = f_A(\lambda) \Rightarrow \text{finished!}$$

$$z_{LD} = 22 \frac{1}{3}$$

RCSP optimal path is  
(1,2,4,7) with cost 24

Duality gap is  $1 \frac{2}{3}$