Exercises on Basic Algorithms
2005 DIKU Summer School on Shortest Paths

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Exercise 1

Prove the Simple Path Theorem: If in the scanning methods applied to a graph with no negative cycles $d(v) < \infty$, then there is a simple s-v path of length $d(v)$.

Exercise 2

If the scanning method is applied to a graph with a negative cycle, than for some vertex v, $d(v)$ is unbounded from below.

Exercise 3

We know that if the scanning method is applied to a graph with a negative cycle, then after some time the graph induced by the parent pointers always has a cycle. Initially, the graph has no cycles (and no arcs). Show that during the intermediate stage, the graph can acquire a cycle and then become acyclic again.

Exercise 4

Pape's algorithm is an implementation of the scaling method using a `dequeue` Q. Vertices are removed from the head of Q. First-time labeled vertices are added to the tail, others to the head. Construct an example on which the algorithm takes exponential time. You may need to use large arc lengths.

Exercise 5:

Pallattino's algorithm uses two FIFO queues, H and L. When a vertex v becomes labeled, it is added to L if v has been unreached and to H is it has been scanned. The next vertex to be scanned is removed from H if it is not empty and from L otherwise. Show that the algorithm runs in $O(nm)$ time.

Exercise 6:

Show that the path fixing procedure of the scaling shortest path algorithm does not create new bad arcs and either finds a negative cycle or fixes all bad arcs on P.

Exercise 7:

Consider the bidirectional Dijkstra's algorithm on a graph with nonnegative arc lengths with the following stopping criteria. Let μ is the length of the shortest path seen so far and let d_f and d_r be minimum distance labels of labeled vertices for the forward and reverse searches, respectively. Stop when $d_f + f_r \geq \mu$.

(a) Prove that the algorithm is correct.

(b) Show that the algorithm scans no more vertices than the standard bidirectional algorithm (that stops as soon as a vertex scanned in one direction is about to be scanned in the other direction).

(c) Give an example when the algorithm scans fewer vertices than the standard one.

Exercise 8:

Suppose p is a feasible potential function (e.g., $\ell_p(v,w) = \ell(v,w) - p(v) + p(w)$) and $p_t = 0$. Prove that for any v, $p(v) \leq dist(v,t)$.

Exercise 9:

Let p_1 and p_2 be feasible potential functions. Show that the minimum, maximum, and average of p_1 and p_2 is feasible.

Exercise 10:

Propose a landmark selection heuristic. Explain why you think it will work well.