

3. PASTA BASTA

The Assignment (June 1992)

An Italian company "Pasta Basta" produces different kinds of pastas and certain meat dishes including a famous lasagne. To ensure sufficient *durability* of the lasagne which mainly is made for export, certain quantities of two different *preservatives* P and Q are added.

Let p and q be the number of units produced per day of preservatives P and Q respectively. A *production plan* (p,q) is a combination of p and q be they integer-valued or not. A *feasible production plan* is a production plan satisfying the following constraints:

At least 1 unit of P must be produced per day.

At most 8 units of P must be produced per day.

At least 1 unit of Q must be produced per day.

At most 3.5 units of Q must be produced per day.

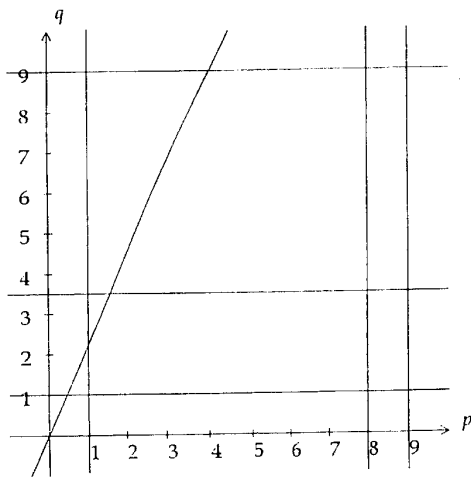
For each unit of P produced, at most 2 units of Q should be produced.

The total production cannot exceed 9 units per day.

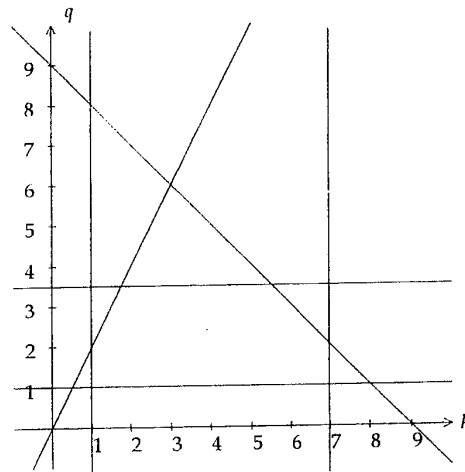
Questions Q1-Q4: The basic model

With each production plan (p,q) corresponds a certain *durability* $d(p,q) = 7p + 11q$ of the final product. The manager of Pasta Basta wishes to find an *optimal production plan*, that is, a feasible production plan which maximises the durability.

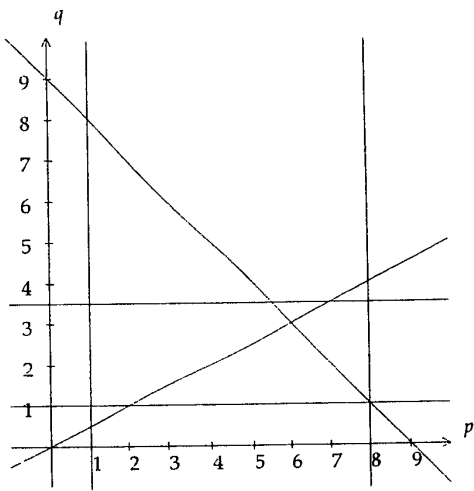
Q1: Which one of the four figures 1a, 1b, 1c, 1d is an adequate graphical representation of the constraints to be observed by the manager?



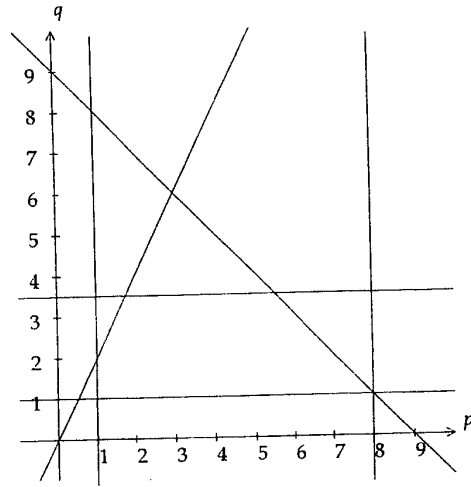
a: Figure 1a



b: Figure 1b



c: Figure 1c



d: Figure 1d

Q2: Are there any redundant constraints?

- a: Yes b: No

Q3: An *iso-durability line* is a line along which all combinations of p and q correspond to the same value of $d(p,q)$. Three different sets (A,B,C) of such *iso-durability lines* are shown in figure 2. Which one of these sets corresponds to the objective function $d(p,q) = 7p + 11q$?

- a: set A b: set B c: set C

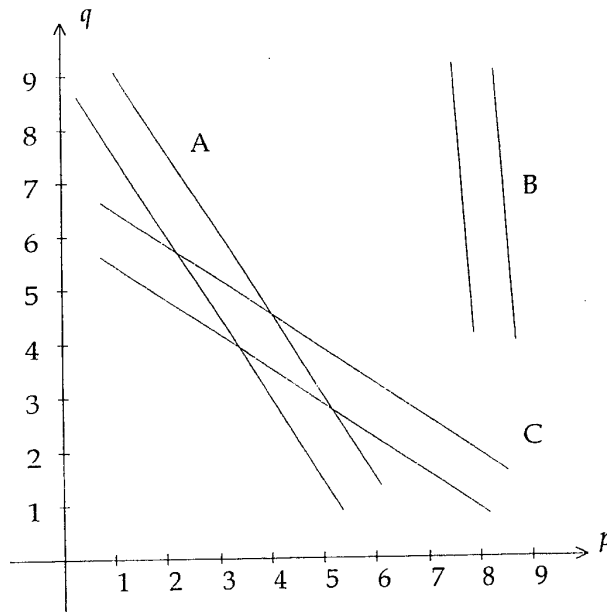


Figure 2

- Q4:** Find an optimal production plan. Does the maximum value of $d(p,q)$ so obtained equal
- a: 71? b: 67? c: 77? d: 81? e: 53?

Questions Q5-Q6: Stability of solution

- Q5:** Replace the objective function $d(p,q) = 7p + 11q$ by $kp + 11q$. What is the largest value of k for which the optimal solution found in Q4 remains optimal?
- a: 9.6 b: 10 c: 10.5 d: 11 e: 11.5
- Q6:** Suppose the requirement (first page), "For each unit of P produced, at most 2 units of Q should be produced" is replaced by: "For each unit of P produced, at most r units of Q should be produced". What is the smallest value of r for which the optimal solution found in Q4 remains optimal?
- a: 4 b: 3.5 c: 35/11 d: 1 e: 7/11 f: 5/11

Questions Q7-Q12: An alternative production process

Due to the preservatives used, a number of consumers appears to be allergic to the Pasta Basta lasagne. These preservatives, however, need not be manufactured as hitherto done but can also, without unpleasant side effects for the consumers, be extracted from fresh fruit. The following table specifies the contents (number of units) of preservatives P and Q that can be extracted from 10,000 kilograms of each of four sorts F1,F2,F3,F4 of fresh fruit:

	F1	F2	F3	F4
Preservative P	2	3	0	5
Preservative Q	3	0	2	4
Cost, ECU/100 kg	13	6	4	12

The European Economic Commission (EEC) asserts that, as of 1994, only fresh fruit must be used. It furthermore specifies that the ratio between preservatives P and Q extracted shall equal 7:2.

To maintain the present production and export of lasagne, Pasta Basta will need exactly 9 units in total, and hence 7 units per day of preservative P and 2 units of preservative Q.

Let f_1, f_2, f_3, f_4 denote the amounts (measured in 10,000 kg) of fruits F1,F2, F3,F4 used per day by Pasta Basta for extracting the necessary quantities of preservatives P and Q. Taking seasonal variation into account, the bottom line of the table above gives the average price per year for buying and storing 100 kilograms of each of the four fruits. Obviously, in his choice of the fruits to be used daily, the manager wishes to minimise these costs.

Q7: Three different formulations of the manager's problem are suggested below. Which one is correct?

a: $\max 13f_1 + 6f_2 + 4f_3 + 12f_4$
 $2f_1 + 3f_2 + 5f_4 = 7; 3f_1 + 2f_3 + 4f_4 = 2;$
 f_1, f_2, f_3, f_4 nonnegative

b: $\min 13f_1 + 6f_2 + 4f_3 + 12f_4$
 $2f_1 + 3f_2 + 5f_4 = 7; 3f_1 + 2f_3 + 4f_4 = 2;$
 f_1, f_2, f_3, f_4 nonnegative

c: $\min 13f_1 + 6f_2 + 4f_3 + 12f_4$
 $2f_1 + 3f_2 + 5f_4 \leq 7; 3f_1 + 2f_3 + 4f_4 \leq 2;$
 f_1, f_2, f_3, f_4 nonnegative

Q8: Based on preservatives extracted from fresh fruit and observing the EEC ratio requirement, what is the durability of the resulting lasagne?

a: 63 b: 67 c: 71 d: 75 e: 79

Q9: It appears from Q7 that the manager's problem is a linear programming problem with two constraints. Unless the feasible region is empty, the problem will then have an optimal solution in which at most two variables are positive.

Given that, no sophisticated technique is necessary to solve the manager's problem to optimality. But first, look at the numbers in the table on the previous page and consider the following question: Is it immediately obvious that a certain fruit will not be used in an optimal solution?

a: Yes, F1 b: Yes, F2 c: Yes, F3 d: Yes, F4 e: No

Q10: Now, in an optimal solution, how many kilograms of fruit F4 will be used per day?

a: 0 b: 10 c: 50 d: 60 e: 100 f: 500 g: 5,000

Q11: In an optimal solution to the manager's problem, the value of the objective function will be 15 (corresponding to 1,500 ECU) and the *shadow prices* y_p, y_q associated with the two restrictions become $y_p = 2$ and $y_q = 0.5$.

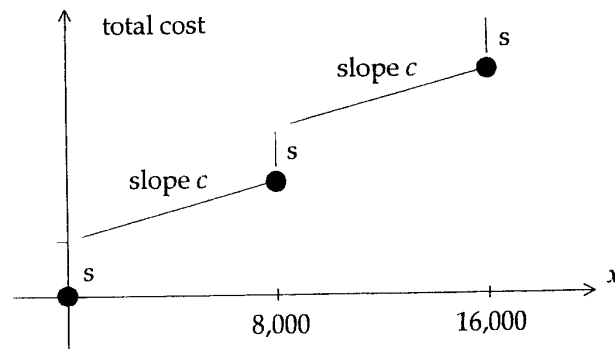
Suppose that "7" is replaced by "7 + k", $k \geq 0$, in the right hand side of the first constraint and that the problem is resolved. Will the resulting value of the objective function then be

a: $15 + 0.5k$? b: $15 + 2k$? c: 15 ?
d: $15 - 0.5k$? e: $15 - 2k$?

Q12: A very complex model is used daily for the general management of Pasta Basta. Only a small part deals with the new production process to be introduced by January 1994 at the latest to comply with the EEC requirement.

Based on the fresh fruit discussed above, the extraction of preservatives P and Q is carried out on a number of identical machines each having a processing capacity of 8,000 kilograms per day. For each machine there is a certain *set-up cost*, $s, s > 0$, to be

paid daily but *only* if the machine actually is used. Furthermore, there is a cost c per kilogram processed. The cost structure can be visualised as follows:



where x is the total number of kilograms processed per day. Three suggestions as to how this cost structure should be modelled are given below. Which one is correct?

a: $\min zs + cx ; 8,000z - x \geq 0, z = 0,1,2,\dots$

b: $\min zs + cx ; z - 8,000x \geq 0, z = 0,1,2,\dots$

c: $\min zs + cx ; 8,000z - x \geq 0, z = 0 \text{ or } 1$

Questions Q13-Q15: Subcontracting

The general image of Pasta Basta and the company's market share abroad are matters of deep concern to its manager. He is therefore determined to launch the new production process a year before it is requested.

Pasta Basta, however, has in addition sufficient resources in 1993 to maintain the current (and profitable) production of preservatives P and Q in the old-fashioned way and initiates negotiations with *Ducolax*, a chemical firm which actually can use preservatives P and Q for some of their non-food products. Throughout all of 1993, Ducolax agrees every day to buy 7 units of preservative P and 2 units of preservative Q at unit prices y_p and y_q (each measured in 100 ECU) respectively provided that y_p and y_q do not exceed the prices at which similar quantities of preservatives P and Q can be bought as contents of fruits F1,F2,F3,F4, not even if the costs of extracting these preservatives are disregarded. In other words, y_p and y_q must be fixed such that they so to speak can compete with each of the four fruits available. At the same time, the Pasta Basta manager wants to maximise his profit.

Q13: The objective function is straightforward: $\max 7y_p + 2y_q$. Four different sets of constraints are proposed below. Which one reflects Pasta Basta's situation correctly?

a: $7y_p + 2y_q \leq 15, y_p \geq 0, y_q \geq 0$

b: $2y_p + 3y_q \leq 13, 3y_p \leq 6, 2y_q \leq 4, 5y_p + 4y_q \leq 12$

c: $2y_p + 3y_q \leq 13, y_p \leq 6, y_q \leq 4, 5y_p + 4y_q \leq 12, y_p \geq 0, y_q \geq 0$

d: $2y_p + 3y_q \geq 13, 3y_p \geq 6, 2y_q \geq 4, 5y_p + 4y_q \geq 12, y_p \geq 0, y_q \geq 0$

Q14: What is the optimal solution to the problem posed in Q13 (hint: no computation is necessary!)?

a: $(y_p, y_q) = (1, 4)$ b: $(y_p, y_q) = (3/7, 6)$

c: $(y_p, y_q) = (2, 0.5)$ d: $(y_p, y_q) = (5/7, 5)$

Q15: In the optimal solution, what is the shadow price associated with the second constraint (still, no computation is necessary)?

- a: 0 b: 0.5 c: 1 d: 1.5 e: 2.0

Questions Q16-Q20: Looking ahead ...

Allergic consumers, the advent of new technologies, recruitment and training of new personnel, inner market deliberations ... lots of pressing issues to keep our Pasta Basta manager and his staff busy. Let us assist them with some of the more tangible problems they are facing.

Questions Q16-Q18: The refrigeration system

To extract the preservatives from fresh fruit is a somewhat complicated affair. Here below is shown a directed network representing the refrigeration system. The network consists of 8 *vertices*: A is the *source* where *flow* is generated, H is the *sink* where flow is absorbed, and the remaining 6 vertices B, C, ..., G are *intermediate* in the sense that *inflow* must equal *outflow*. The vertices are connected by directed edges representing *pipe sections* as shown. With each edge is associated a number representing the *capacity* measured in litres per second. The flow itself is a certain refrigerant known as VX4 which is pumped through the network. To exemplify some of these notions: irrespective of the pumps used, at most 6 litres per second of VX4 can flow in pipe section BC from B to C.

