Written exam, 27 January 2006

Alle hjælpemidler (bøger, noter, m.v., men *ikke* lommeregner) er tilladte. Det er desuden tilladt at skrive med blyant, hvis resultatet bliver klart læseligt.

YOUR ASSIGNMENT:

20 questions **Q1-Q20** are posed on the subsequent pages.

Q1-Q8, and **Q11-Q18** are *multiple choice questions*. For each of these, the only correct answer is one of the answers proposed. To answer a specific question, you are requested without further explanation *clearly* to write, for example, **"7B"** as your answer to question Q7.

Q9-Q10 and **Q19-Q20** are ordinary *text questions*.

Each correct answer to a
- multiple choice question gives 4 points
- text question gives 9 points

The maximum score is thus 100 points.

Linear Programming (Q1-Q7)

Let **P**($\alpha,\beta,\gamma,\Delta$) be the family of LP instances defined by

P($\alpha,\beta,\gamma,\Delta$):

$$\max w = 6\alpha x_1 + \beta x_2 - 8x_3$$

$$3x_1 + x_2 + (1/\gamma)x_3 = \Delta$$

$$x_1, x_2, x_3 \text{ nonnegative}$$

where $\alpha,\beta,\gamma,\Delta$, $\gamma\neq 0$, are given constants to be viewed as *parameters*. Thus, e.g., **P(4,8,1,3)** is the instance obtained by substituting 4,8,1,3 for $\alpha,\beta,\gamma,\Delta$ respectively.

Today's best clue before you proceed: tedious simplex iterations are nowhere necessary!

Q1: Solve **P(4, 8, 1, 3)** to optimality. What is the number of *optimal basic solutions*?

1A) 0 1B) 1 1C) 2 1D) 3

Q2: What is the number of *optimal solutions* to **P(0, 2, -1/8, 0)**?

2A) 0 2B) 1 2C) 2 2D) 3

Let **D**($\alpha,\beta,\gamma,\Delta$) be the dual of **P**($\alpha,\beta,\gamma,\Delta$). Whenever **D**($\alpha,\beta,\gamma,\Delta$) is considered in the sequel, make sure that the dual constraint involving γ is correctly formulated.

D(α, **2, -1/8,** Δ) is the subfamily of **D**($\alpha,\beta,\gamma,\Delta$) where α and Δ can take any value.

Q3: What is the number of *basic feasible solutions* to **D**(α, **2, -1/8,** Δ)?

3A) 0 3B) 1 3C) 2 3D) 3

Q4: Unless infeasible or unbounded, solve **D(-6, -10, 1, 3)** to optimality. Let Σ be the *sum* of the dual variables in the solution found. Which one of the following statements is true,

4A) **D(-6, -10, 1, 3)** is infeasible or unbounded

4B) $\Sigma = 0$ 4C) $\Sigma > 0$ 4D) $\Sigma = -8$

For $\delta \geq 0$, let w(δ) be the value of the objective function in an optimal solution to **P(-6,-10, 1, 3+**δ**)**.

Q5: What is the value of w(δ)-w(0),

5A) 0 5B) 8δ 5C) -8δ 5D) 3 5E) something else

Assume that **P**($\alpha,\beta,\gamma,\Delta$) is modified such that x_2 and x_3 no longer are restricted to be nonnegative but can take any value and, hence, are referred to as *free*. Let **PF**($\alpha,\beta,\gamma,\Delta$) be the resulting family of LP instances.

Q6: For $\gamma < 0$ and $\Delta > 0$, how many of the following statements are true,

PF($\alpha,\beta,\gamma,\Delta$) is unbounded or infeasible if $\beta > 8\gamma$

PF($\alpha,\beta,\gamma,\Delta$) is unbounded or infeasible if $\beta = 8\gamma$

There exists values of α,Δ such that **PF**($\alpha,\beta,\gamma,\Delta$) is optimally solvable if $\beta = -8\gamma$

6A) 0 6B) 1 6C) 2 6D) 3

Assume that the four parameters $\alpha,\beta,\gamma,\Delta$ all take the same value called *k*, k>0. Furthermore, recall that a constraint is called *redundant* if its removal leaves the feasible solution space unaltered.

Q7: What is the number of redundant constraints in **D(k,k,k,k)** ?

7A) 0 7B) 1 7C) 2 7D) 3 7E) 4

Integer Programming (Q8)

For the original family **P**($\alpha,\beta,\gamma,\Delta$) of LP instances, we add the requirement that all three variables should be *integer-valued*.

Q8: For which values of k, k>0, will *all* variables in *any basic solution* to P(k,k,k,k) be integer-valued,

8A) k=1 8B) k=2 8C) k is divisible by 3

8D) k is divisible by 5 8E) no such values of k exists

The Transportation Model (Q9-Q10)

The New Year's party at *Booze, Inc.* was pretty wild. Among the many memorable events was the company's sole OR analyst caught in an intimate situation with the manager's wife and sacked the day after. The *Transportation Model (TP-Booze)*, of vital importance to the company's daily operations, was afterwards found as shredded paper in his abandoned office.

Using the notation of [Taha] (see e.g. Table 5.25 [Taha, p. 187]), the sad remainders of the *optimal* solution to *TP-Booze* is exhibited below. The majority of the 5×4 entries show the value of c_{ij} where M is a large number, sufficiently large to ensure that the corresponding $u_i + v_j - c_{ij}$ is *negative* for any finite values of u_i and v_j. The three numbers shown in parantheses are the only known values of x_{ij}:

u_i, v_j	$v_1 = 3$	$v_2 = ?$	$v_3 = -2$	$v_4 = 5$	**Supply**
$u_1 = 0$	5	6 (1)	0	5	$a_1 = 5$
$u_2 = 2$	5	M	M	$c_{24} = ?$ (4)	$a_2 = 7$
$u_3 = 7$	13	15	5	16	$a_3 = 2$
$u_4 = 4$	8	10	2	11	$a_4 = 8$
$u_5 = ?$	M	$c_{52} = ?$ (0)	14	M	$a_5 = ?$
Demand	$b_1 = ?$	$b_2 = 6$	$b_3 = ?$	$b_4 = ?$	

TP-Booze is *balanced.* Yet, quite a lot is needed to reconstruct the full table. Let this challenging task be split into two parts:

Q9 ***(text question):***

What are the values of b_1, b_4, c_{24}, and v_2 ?

A bright accountant recalls that the objective function value in an optimal solution is 163.

Q10 ***(text question):***

i) Complete the solution in terms of x_{ij}, a_5, and the missing demands.

ii) Will the solution remain optimal if $c_{52} = 23$?