Institut for Matematiske Fag, Københavns Universitet, Exam in Operations Research, June 28, 2006

There are 3 exercises on 3 pages (and one extra, if time permits). The exam is three hours. It is allowed to use a pencil. Books and notes are allowed, but not a pocket calculator or a computer.

Exercise 1 (*weight 40%*)

Consider the linear program (P)

$$
\begin{aligned}
&\text{Maximize } Z = 3x_1 + 5x_2 + 7x_3 \\
&\quad\text{subject to} \\
&2x_1 + 3x_2 + x_3 \leq 8 \\
&x_1 + 2x_2 + 2x_3 \leq 6 \\
&3x_1 + 5x_2 + 4x_3 \leq 15 \\
&x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
\end{aligned}
$$

Let x_4, x_5 and x_6 be the slack variables in the three first constraints. Consider the basis $B = \{1, 2, 3\}$. Observe that

$$
A_B = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 3 & 5 & 4 \end{pmatrix} \text{ and } A_B^{-1} = \begin{pmatrix} -2 & -7 & 4 \\ 2 & 5 & -3 \\ -1 & -1 & 1 \end{pmatrix}.
$$

(i) Formulate the dual of (P).

(ii) Compute the primal and dual basic solutions corresponding to B. Conclude that B is feasible for (P), but not for (D). What are the reduced costs corresponding to B?

(iii) Construct the simplex tableau associated with the basis B.

(iv) Perform two pivots from the basis B. The resulting basis should be an optimal basis.

(v) Consider changing the right-hand-sides from $b = (8, 6, 15)^T$ to $b = (8, 6, 15 - \delta_1)^T$, where δ_1 is some number. For which values of δ_1 does the basis found in (iv) remain optimal?

(vi) Consider changing the objective function from $c = (3, 5, 7)^T$ to $b = (3, 5 + \delta_2, 7)^T$, where δ_2 is some number. For which values of δ_2 does the basis found in (iv) remain optimal?

Exercise 2 (*weight 20%*)

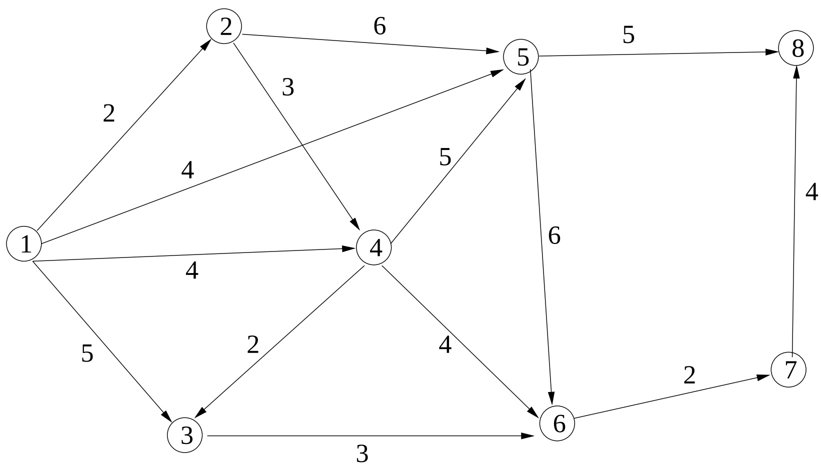

Figure 1: A directed graph with edge lengths

Consider the directed graph in Fig. 2.

(i) Find a shortest path from node 1 to node 8 in G by using Dijkstra's method. Please explain every step.

(ii) Formulate the problem of finding a shortest path from node 1 to node 8 in G as a linear program.

Exercise 3 (*weight 40%*)

	1	2	3	4	Supply(s_i)
1	8	15	12	16	30
2	14	9	7	11	25
3	12	14	10	16	40
4	6	12	14	8	45
Demand(d_j)	35	35	50	20	

Table 1:

Consider the transportation problem with the data in Table 1.

(i) Find a basic feasible solution for the transportation problem of Table 1 by using the North-West corner rule.

(ii) Present the resulting basic solution in a transportation tree.

(iii) Compute the corresponding dual basic solution.

(iv) Find a variable with a negative reduced cost, and draw the corresponding cycle obtained.

(v) Find the new basic feasible solution obtained by adding this variable to the basis.

(vi) Perform one more pivot of the transportation method by following steps (ii)-(v).

Exercise 4 (*extra*)

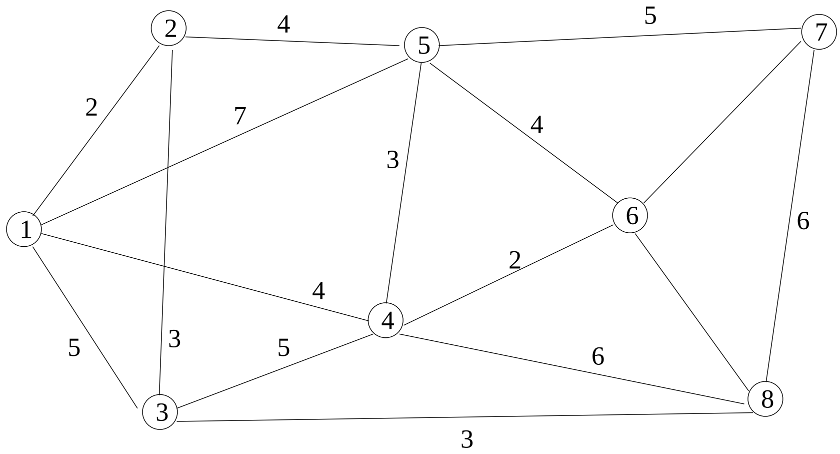

Figure 2: A graph G with edge weights

Consider the graph G in Fig. 1. Find a minimum weight spanning tree of the graph G by using Prim's method. Please explain every step.