Decomposition of the Capacitated Lot Sizing Problem with Setup Times

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Overview

- Short recap of Column Generation

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- Notation and Mathematical model for CLST

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- Capacitated Lot Sizing Problem with Setup Times (CLST) example

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- Decomposition of Network reformulation for CLST

CLST Big Bucket Model

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- In a big bucket model scheduling is not included in contrast to the small bucket model
- Setup times indicate the time it takes to prepare the machine to the next item.
- We also consider holding cost, setup cost, variable production cost and variable production time for each product

Notation

N set of items $\{1, .., n\}$

T set of time periods $\{1, ..., m\}$

d_{it} demand in period t for item i

vc_{it} production cost in period t for item i

hc_{it} holding cost in period t for item i

sc_{it} setup cost in period t for item i

st_{it} setup time for item i in period t

sd_{itk} sum of demand from period i to period k

vt_{it} production time for item i in period t

fc_i cost of initial inventory for item i

Notation Continued

cap_t capacity in period t

cap_{it} capacity for item i in period t.
$cap_{it} = \min\{\frac{cap_t}{vt_{it}} - \frac{st_{it}}{vt_{it}}, sd_{itm}\}$

x_{it} production in period t for item i

y_{it} Indicates if machine is setup for item i in

s_{it} stock in period t for item i

si_i initial stock of item i period t

Mathematical model

$$\min \sum_{i\in P} fc_i si_i + \sum_{i\in P}\sum_{t\in T} vc_{it}x_{it} + hc_{it}s_{it} + sc_{it}y_{it}$$

$$
\begin{array}{llll}
si_i + x_i, 1 & = d_{1,i} + s_{i,1} & \forall i \in P & \\
s_{i,t-1} + x_{it} & = d_{it} + s_{it} & \forall i \in P & \forall t \in T \backslash \{1\} \\
x_{it} & \leq cap_{it} & \forall i \in P & \forall t \in T \\
\sum_{i\in P} st_{it}y_{it} + vt_{it}x_{it} & \leq cap_t & \forall t \in T & \\
\multicolumn{2}{l}{x_{it} \geq 0, s_{it} \geq 0, si_i \geq 0, y_{it} \in \{0,1\}} & \forall i \in P & \forall t \in T
\end{array}
$$

Example of CLST

Instance with 5 periods and 2 items and capacity of 10 in each period. We disregard the production, setup and holding cost and just look for a solution with no initial inventory:

$item1$						$item2$					
$period$	1	2	3	4	5	$period$	1	2	3	4	5
d	3	2	2	3	4	d	0	5	2	3	4
st	2	2	2	2	2	st	2	2	2	2	2
vt	1	1	1	1	1	vt	1	1	1	1	1
fc	10	10	10	10	10	fc	10	10	10	10	10

A solution to the example

	$item1$					$item2$				
$period$	1	2	3	4	5	1	2	3	4	5
x	8	0	6	0	0	0	8	0	6	0
s	5	3	7	4	0	0	3	1	4	0
st	2	0	2	0	0	0	2	0	2	0

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- A solution is a combination of initial inventory satisfying demand in period t to k and production in period t satisfying demand from period t to k
- Idea is to find new formulation where we formulate problem as a combination of production schedules that covers a given set of time periods for each item.
- Note that Wagner Whitin property does not hold.

Reformulation continued

Reformulation continued

The total production and holding cost for producing item i in period t to satisfy demand from period t to period k

$$cv_{itk} = vc_{it} sd_{itk} + \sum_{s=t+1}^{k} \sum_{u=t}^{s-1} hc_{iu} d_{is}$$

Reformulation continued

The total production and holding cost for producing item i in period t to satisfy demand from period t to period k

$$cv_{itk} = vc_{it} sd_{itk} + \sum_{s=t+1}^{k} \sum_{u=t}^{s-1} hc_{iu} d_{is}$$

The total production and holding cost for initial inventory for item i to satisfy demand from period t to period k

$$ci_{it} = fc_i sd_{i1t} + \sum_{s=2}^{t} \sum_{u=1}^{s-1} hc_{iu} d_{is}$$

Reformulation continued

zv_{itk} The fraction of production plan for product i in period t that satisfies demand from period t to k.

w_{it} Be the fraction of initial inventory plan for product i where demand is satisfied for the first t periods.

Network Reformulation model

$$\min \sum_{i\in P}\sum_{t\in T} sc_{it}y_{it} + ci_{it}w_{it} + \sum_{k=t}^{m} cv_{itk}zv_{itk}$$

$$\sum_{k\in T} w_{ik} + zv_{i,1,k} = 1 \qquad \forall i \in P$$

$$w_{i,t-1} + \sum_{k=1}^{t-1} zv_{ik,t-1} - \sum_{k=t}^{m} z_{ik,t-1} = 0 \qquad \forall i \in P \quad \forall t \in T\backslash 1$$

$$\sum_{k=t}^{m} zv_{itk} \leq y_{it} \qquad \forall i \in P, \quad \forall t \in T$$

$$\sum st_{it}y_{it} + \sum_{i\in P}\sum_{k=t}^{m} vt_{it}sd_{itk}z_{itk} \leq cap_t \qquad \forall t \in T$$

$$y_{it} \in \{0,1\}, \quad w_{it} \geq 0 \qquad \forall i \in P, \quad \forall t \in T$$

Reformulation example

$item1$						$item2$					
$period$	1	2	3	4	5	$period$	1	2	3	4	5
d	3	2	2	3	4	d	0	5	2	3	4
st	2	2	2	2	2	st	2	2	2	2	2
vt	1	1	1	1	1	vt	1	1	1	1	1
fc	10	10	10	10	10	fc	10	10	10	10	10

Example of objective

ci_{11}

ci_{12}

ci_{13}

ci_{14}

ci_{15}

Example of objective

$$
\begin{aligned}
ci_{11} &= 10 * 3 + 0 = 30 \\
ci_{12} & \\
ci_{13} & \\
ci_{14} & \\
ci_{15} &
\end{aligned}
$$

Example of objective

$$
\begin{aligned}
ci_{11} &= 10 * 3 + 0 = 30 \\
ci_{12} &= 10 * 5 + 0 = 50 \\
ci_{13} & \\
ci_{14} & \\
ci_{15} &
\end{aligned}
$$

Example of objective

$$\begin{aligned} ci_{11} &= 10 * 3 + 0 = 30 \\ ci_{12} &= 10 * 5 + 0 = 50 \\ ci_{13} &= 10 * 7 + 0 = 70 \\ ci_{14} &= 10 * 10 + 0 = 100 \\ ci_{15} &= 10 * 14 + 0 = 140 \end{aligned}$$

Network reformulation solution

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$$z_{115} = \frac{1}{7}$$

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$$
\begin{aligned}
z_{115} &= \frac{1}{7} \\
z_{112} &= \frac{6}{7}
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$$

Network reformulation solution

$$
\begin{aligned}
z_{115} &= \frac{1}{7} \\
z_{112} &= \frac{6}{7} \\
z_{125} &= \frac{6}{7}
\end{aligned}
$$

Network reformulation solution

$$
\begin{aligned}
z_{115} &= \frac{1}{7} \\
z_{112} &= \frac{6}{7} \\
z_{125} &= \frac{6}{7} \\
\\
z_{211} &= 1
\end{aligned}
$$

Network reformulation solution

$$\begin{aligned} z_{115} &= \frac{1}{7} \\ z_{112} &= \frac{6}{7} \\ z_{125} &= \frac{6}{7} \end{aligned}$$

$$\begin{aligned} z_{211} &= 1 \\ z_{225} &= \frac{1}{7} \end{aligned}$$

Network reformulation solution

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\begin{aligned}
z_{115} &= \frac{1}{7} \\
z_{112} &= \frac{6}{7} \\
z_{125} &= \frac{6}{7}
\end{aligned}
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$$
\begin{aligned}
z_{211} &= 1 \\
z_{225} &= \frac{1}{7} \\
z_{223} &= \frac{6}{7}
\end{aligned}
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Network reformulation solution

$$\begin{aligned} z_{115} &= \frac{1}{7} \\ z_{112} &= \frac{6}{7} \\ z_{125} &= \frac{6}{7} \end{aligned}$$

$$\begin{aligned} z_{211} &= 1 \\ z_{225} &= \frac{1}{7} \\ z_{223} &= \frac{6}{7} \\ z_{245} &= \frac{6}{7} \end{aligned}$$

Decomposition

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We consider the m polytopes:

$$X^t = \begin{cases} \sum_{k=t}^{m} zv_{itk} & \leq y_{it} \quad \forall i \in P, \forall t \in T \\ \sum st_{it} y_{it} + \sum_{i \in P} \sum_{k=t}^{m} vt_{it} sd_{itk} z_{itk} & \leq cap_t \quad \forall t \in T \end{cases}$$

Decomposition continued

S_t Set of production plans that are extreme points in X^t

ct_{tq} Cost of production plan $q \in S_t$

zt_{tq} decision variable assosiated with schedule q

a_{iktq} Indicates how much variable z_{itk} is set in extreme point $q \in S_t$

Master Problem

$$\min \sum_{i \in P} \sum_{t \in T} ci_{it} w_{it} + \sum_{t \in T} \sum_{q \in S_t} ct_{tq} zt_{tq}$$

$$\sum_{k=t}^{m} zv_{itk} \leq y_{it} \quad \forall i \in P, \quad \forall t \in T$$

$$w_{i,t-1} + \sum_{k=1}^{t-1} \sum_{q \in S_k} a_{ik,t-1,q} zt_{tq} - \sum_{k=1}^{m} \sum_{q \in S_t} a_{itkq} zt_{tq} = 0 \quad \forall i \in P \quad \forall t \in T \backslash \{1\}, (\pi_{it})$$

$$\sum_{q \in S_t} zt_{tq} = 1 \quad \forall t \in T, (\mu_t)$$

$$zt_{tq} \geq 0$$

Reduced Cost

$a_{itkq} = zv_{itk}$

$ct_{tq} = \sum_{i \in P} sc_{it} + \sum_{i \in P} \sum_{k=t}^{m} cv_{itk} zv_{itk}$

Reduced Cost

$a_{itkq} = zv_{itk}$
$ct_{tq} = \sum_{i \in P} sc_{it} + \sum_{i \in P} \sum_{k=t}^{m} cv_{itk} zv_{itk}$ The reduced cost for a column where $t \neq m$ then becomes:

$$\hat{ct}_{tq} = \sum_{i \in P} sc_{it} y_{it} + \sum_{i \in P} \sum_{k=t}^{m} cv_{itk} zv_{itk} - \sum_{i \in P} \sum_{k=t}^{m} \pi_{it} zv_{itk} + \sum_{i \in P} \sum_{k=t}^{m-1} \pi_{i,k+1} zv_{itk} - \mu_t$$

Reduced Cost

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$ct_{tq} = \sum_{i \in P} sc_{it} + \sum_{i \in P} \sum_{k=t}^{m} cv_{itk} zv_{itk}$ The reduced cost for a column where $t \neq m$ then becomes:

$$\begin{aligned}
\hat{ct_{tq}} &= \sum_{i \in P} sc_{it} y_{it} + \sum_{i \in P} \sum_{k=t}^{m} cv_{itk} zv_{itk} - \sum_{i \in P} \sum_{k=t}^{m} \pi_{it} zv_{itk} + \\
&\sum_{i \in P} \sum_{k=t}^{m-1} \pi_{i,k+1} zv_{itk} - \mu_t \\
&= \sum_{i \in P} sc_{it} y_{it} + \sum_{i \in P} \sum_{k=t}^{m-1} (cv_{itk} - \pi_{it} - \pi_{i,k+1}) zv_{itk}
\end{aligned}$$

Pricing Problem

$$\min \sum_{i \in P} sc_{it} y_{it} + \sum_{i \in P} \sum_{k=t}^{m-1} (cv_{itk} - \pi_{it} - \pi_{i,k+1}) zv_{itk}$$

$$\sum_{k=t}^{m} zv_{itk} \leq y_{it} \qquad \forall i \in P, \forall t \in T$$

$$\sum st_{it} y_{it} + \sum_{i \in P} \sum_{k=t}^{m} vt_{it} sd_{itk} z_{itk} \leq cap_t \qquad \forall t \in T$$

$$y_{it} \in \{0,1\}, \quad zv_{itk} \geq 0 \qquad \forall i \in P, k \in T$$