

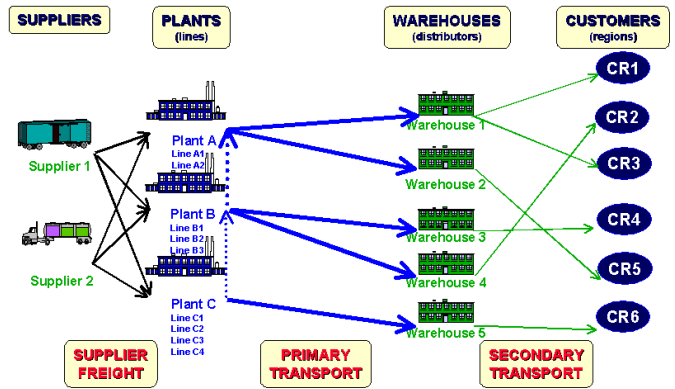
# Capacity Planning

## Sales and Operations Planning

- Forecasting
- Capacity planning
- Inventory optimization
- How much capacity assigned to each production unit?
- Realistic capacity estimates
- Strategic level
- Moderately long time horizon
- Larger time buckets
- Data not known exactly

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# Operations Planning



- Pochet-Wolsey's model is too large, NP-hard
- Decomposition → inaccurate model
- Medium-term planning horizon, strategic level
- Larger time buckets

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## Some definitions

- *Work in Process* inventory between the start and end points of a product routing is called work in process (WIP)
- *WIP inventory* work in process inventory
- *FGI inventory* final goods inventory
- *Lead time* of a given routing or line is the time allotted for production of a part on that routing or line.
- *Little's Law*

$$\text{WIP} = \text{Throughput} \times \text{cycle time}$$



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## Master Production Scheduling Model (Pochet-Wolsey)

- time horizon  $t = 1, \dots, n$
- $\phi_t^i$  unit production cost
- $q_t^i$  fixed production cost
- $\pi_t^i$  inventory cost
- $D_t^i$  demand

### Decision variables

- $x_t^i$  production lot size in period  $t$
- $y_t^i$  binary variable indicating production period  $t$
- $I_t^i$  inventory at end of period  $t$

### Moreover

- $C_t^k$  available capacity, resource  $k$  time  $t$
- $\xi^{ik}$  per unit resource consumption
- $\beta^{ik}$  fixed resource consumption

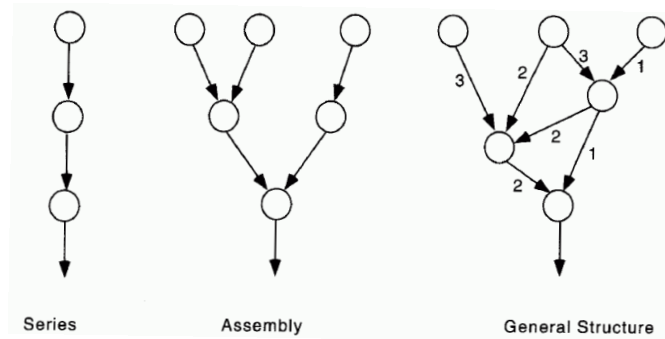
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## Material Requirement Planning Model (Pochet-Wolsey)

Multi-item multi-level capacitated lot-sizing model

- Optimize simultaneously production and purchase of all items
- from raw materials to finished products
- satisfy external demands
- satisfy internal demands
- short-term horizon

Bill of materials (BOM)



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## Material Requirement Planning Model (Pochet-Wolsey)

- $S(i)$  set of direct successors of  $i$  (items consuming  $i$ )
- $r^{ij}$  amount of item  $i$  required to make one item  $j$
- $\gamma^i$  lead-time to produce or deliver a lot of  $i$   
i.e.  $x_t^i$  can be delivered at time  $t + \gamma^i$

Model

$$\min \sum_{i=1}^m \sum_{t=1}^n \phi_t^i x_t^i + q_t^i y_t^i + \pi_t^i I_t^i$$

Subject to:

$$I_{t-1}^i + x_{t-\gamma_t}^i = \left( D_t^i + \sum_{j \in S(i)} r^{ij} x_t^j \right) + I_t^i \quad \forall i, t$$

$$x_t^i \leq M_t^i y_t^i \quad \forall i, t$$

$$\sum_{i=1}^m \xi^{ik} x_t^i + \sum_{i=1}^m \beta^{ik} y_t^i \leq C_t^k \quad \forall t$$

$$x_t^i \in \mathbb{R}_+, I_t^i \in \mathbb{R}_+, y_t^i \in \{0, 1\}$$

Large, NP-hard model, difficult to solve

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## Capacity of Resources

- Gross capacity
- Usable capacity
- Productivity factor

Resource Description	Gross Capacity (hours/day)	Productivity Factor	Usable Capacity (hours/day)
Machine S100	8	0.95	7.6
Forklift	8	0.85	6.8
Machine ASS	16	0.85	13.6
Machine PPP	8	0.95	7.6

Capacity limit is soft (queuing theory)

Capacity depends on  
- variation in arriving jobs  
- variation in lead times

Lead time depends on load

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## Asmundsson, Single-stage multi-product planning

Simplified model

$$\min \sum_i \sum_t \phi_{it} X_{it} + \pi_{it} I_{it}$$

Subject to:

$$I_{it} = I_{i,t-1} + X_{i,t-L} - D_{it} \quad \forall i, t$$

$$\sum_i \xi_{it} X_{it} \leq C_t \quad \forall t$$

$$X_{it}, I_{it} \in \mathbb{R}_+ \quad \forall i, t$$

- LP-model (easy to solve)
- Setup times are neglected (future work)
- Constant lead time (production time)  $L$

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## Introduction

Accurate measurements of manufacturing capacity is hard to obtain (Elmaghraby 1991)

- Stochastic performance analysis models  
Queuing models capture important aspects (Hopp, Spearman 2000)  
Stochastic model of whole production unrealistic
- Deterministic techniques  
Divide planning horizon in discrete buckets  
Assign capacity in each bucket  
Solution satisfying aggregated constraint not feasible in practice
- Integrating approaches (Hung, Leachman 1996)  
Solve LP model for production planning  
Feed into simulator to estimate lead times  
Add cut if not feasible

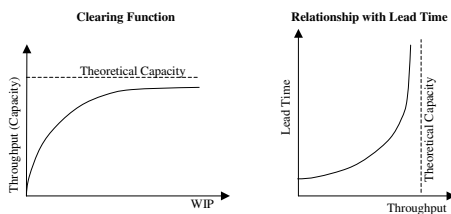


Figure 1 Relationships between throughput, WIP and lead time (Karmarkar 1989)

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## Relation between WIP and system throughput

- Analytic congestion model for closed production systems (Spearman 1991)
- Queuing models of manufacturing systems (Buzacott, Shanthikumar 1993)

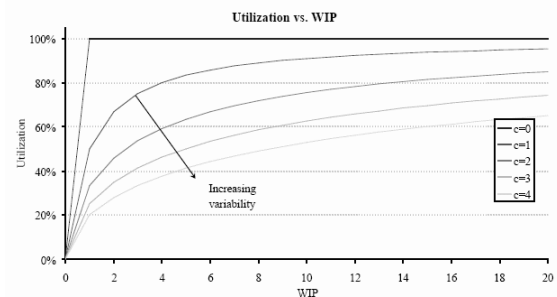
Connection between WIP, variation, utilization (Medhi 1991)

$$WIP = \frac{c^2 \rho^2}{1 - \rho} + \rho$$

$c$  variation service time and arrivals,  $\rho$  utilization of server

$$\rho = \frac{\sqrt{(WIP + 1)^2 + 4WIP(c^2 - 1)} - (WIP + 1)}{2(c^2 - 1)}$$

Figure for different values of  $c$

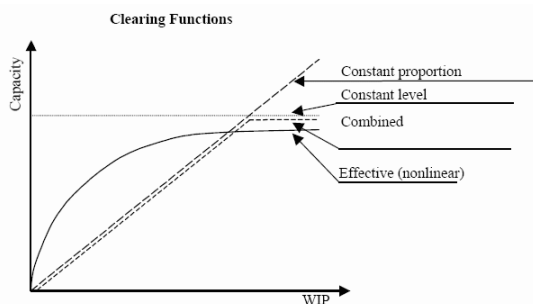


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## Clearing Function

Capacity limit depends on Work-in-process ( $W$ )

$$\sum_i \xi_{it} X_{it} \leq f_t(\sum_i \xi_{it} W_{it})$$



If capacity depends on WIP we need to have cost associated with WIP (internal inventory)

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## Asmundsson

Single Stage Multi-product Planning Model Using Clearing Functions

- $X_{it}$  number of units of product  $i$  produced time  $t$
- $R_{it}$  number of units of product  $i$  released into the stage at the beginning of time period  $t$
- $W_{it}$  number of units of product  $i$  in WIP inventory at the end of period  $t$
- $I_{it}$  number of units of product  $i$  in finished goods inventory at the end of period  $t$
- $\xi_{it}$  amount of resource (machine time) required to produce one unit of item  $i$  time  $t$
- $f_t(W)$  clearing function representing resource in period  $t$
- $D_{it}$  the demand for product  $i$  in period  $t$

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## CF model (Clearing Function)

$$\min \sum_t \sum_i \phi_{it} X_{it} + \omega_{it} W_{it} + \pi_{it} I_{it} + \rho_{it} R_{it}$$

Subject to:

$$W_{it} = W_{i,t-1} - X_{it} + R_{it} \quad \forall i, t$$

$$I_{it} = I_{i,t-1} + X_{it} - D_{it} \quad \forall i, t$$

$$\sum_i \xi_{it} X_{it} \leq f_i \left( \sum_i \xi_{it} W_{it} \right) \quad \forall t$$

$$X_{it}, W_{it}, I_{it}, R_{it} \in \mathbb{R}_+ \quad \forall i, t$$

where

- $\phi_{it}$  cost of producing unit  $i$  time  $t$
- $\omega_{it}$  cost of WIP of unit  $i$  time  $t$
- $\pi_{it}$  cost of inventory unit  $i$  time  $t$
- $\rho_{it}$  cost of releasing unit  $i$  time  $t$

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## Problem with capacity constraint

$$X_A + X_B \leq f(W_A + W_B)$$

for two products  $A, B$

- Solution exists  $X_A > 0, X_B = 0, W_A = 0, W_B > 0$
- Maintain high  $W_B$  if cheap
- Create capacity for  $X_A$

No link between WIP available and production

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## Solution

$Z_{it} \geq 0$  represent an allocation of expected throughput among different products

$$X_{it} \leq Z_{it} f_i \left( \sum_i \xi_{it} W_{it} \right) \quad \forall i, t$$

$$\sum_i Z_{it} = 1 \quad \forall t$$

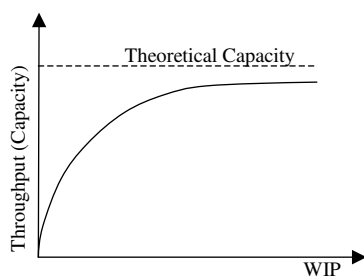
Depends on total WIP.

Prefer WIP of specific product  $i$ .

Assuming  $\xi_{it} W_{it} = Z_{it} \sum_i \xi_{it} W_{it}$  we get (?)

$$X_{it} \leq Z_{it} f_i \left( \sum_i \frac{\xi_{it} W_{it}}{Z_{it}} \right) \quad \forall i, t$$

$$\sum_i Z_{it} = 1 \quad \forall t$$



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## ACF model (Allocated Clearing Function)

$$\min \sum_t \sum_i \phi_{it} X_{it} + \omega_{it} W_{it} + \pi_{it} I_{it} + \rho_{it} R_{it}$$

Subject to:

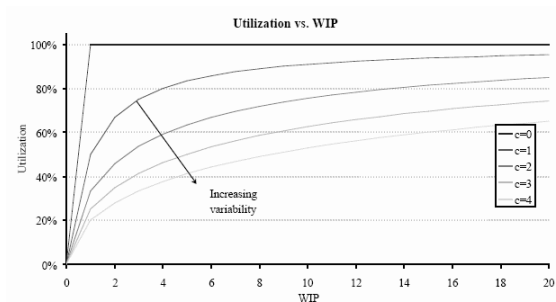
$$W_{it} = W_{i,t-1} - X_{it} + R_{it} \quad \forall i, t$$

$$I_{it} = I_{i,t-1} + X_{it} - D_{it} \quad \forall i, t$$

$$X_{it} \leq Z_{it} f_i \left( \sum_i \frac{\xi_{it} W_{it}}{Z_{it}} \right) \quad \forall t$$

$$\sum_i Z_{it} = 1 \quad \forall t$$

$$X_{it}, W_{it}, I_{it}, R_{it} \in \mathbb{R}_+ \quad \forall i, t$$



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## Outer approximation of clearing function

Outer linear functions  $\alpha^c W_t + \beta^c$  for  $c = 1, \dots, C$

$$\sum_i \xi_{it} W_{it} \leq \alpha^c W_t + \beta^c \quad \forall c, t$$

then

$$f(W) = \min_{c=1, \dots, C} \{ \alpha^c W_t + \beta^c \}$$

Assume that

$$\alpha^1 > \alpha^2 > \alpha^3 > \dots > \alpha^c = 0$$

### Linear model

$$\min \sum_t \sum_i \phi_{it} X_{it} + \omega_{it} W_{it} + \pi_{it} I_{it} + \rho_{it} R_{it}$$

Subject to:

$$W_{it} = W_{i,t-1} - X_{it} + R_{it} \quad \forall i, t$$

$$I_{it} = I_{i,t-1} + X_{it} - D_{it} \quad \forall i, t$$

$$\sum_i \xi_{it} X_{it} \leq \alpha^c \xi_{it} W_{it} + Z_{it} \beta^c \quad \forall i, t, c$$

$$\sum_i Z_{it} = 1 \quad \forall t$$

$$Z_{it}, X_{it}, W_{it}, I_{it}, R_{it} \in \mathbb{R}_+ \quad \forall i, t$$

Which is correct since

$$\begin{aligned} Z_{it} f\left(\frac{\xi_{it} W_{it}}{Z_{it}}\right) &= Z_{it} \min_c \left\{ \alpha^c \frac{\xi_{it} W_{it}}{Z_{it}} + \beta^c \right\} \\ &= \min_c \{ \alpha^c \xi_{it} W_{it} + \beta^c Z_{it} \} \end{aligned}$$

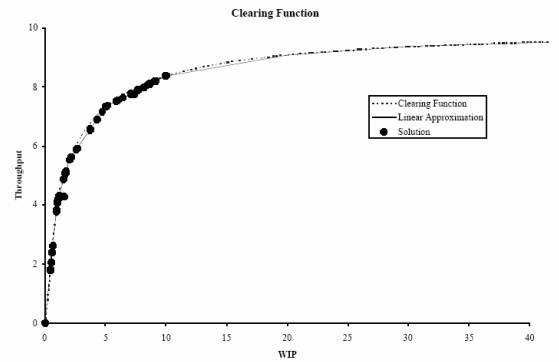
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## Asmundsson, experimental results

Single-stage system, 3 products

- Variation of arrival 0.5 variation of service 2
- Max throughput 10 items / period
- Minimize inventory holding costs  $(\pi_1, \pi_2, \pi_3) = (1, 2, 3)$
- Holding costs of WIP and FGI are same

Clearing function, linear approximation



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## Asmundsson, experimental results

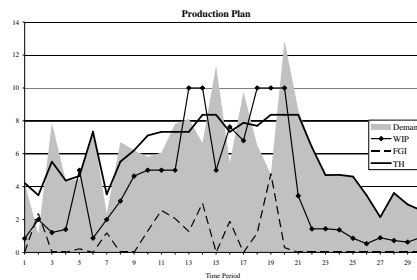


Figure 2 Production plan

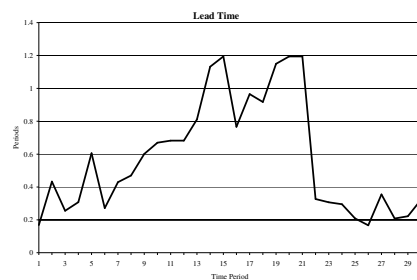


Figure 3 Production lead time

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## Asmundsson, experimental results

Throughput

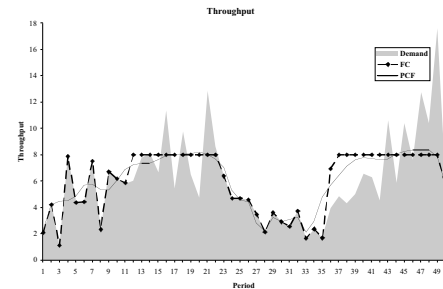


Figure 5: Throughput and Demand of ACF and FC Models.

Demand (gray), throughput from ACF more smooth than FC

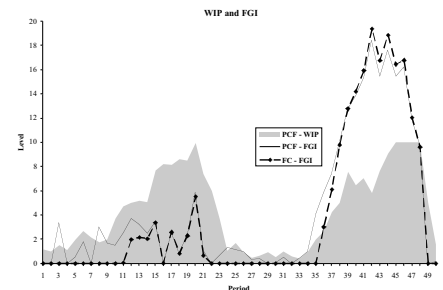


Figure 6: WIP and FGI levels, Cumulative across Items.

WIP high when throughput high, FC model does not capture WIP

FC solution only optimizes FGI holding costs, fails to capture tradeoffs WIP, FGI

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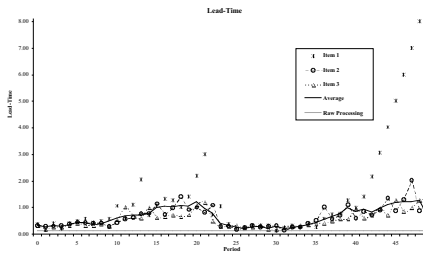


Figure 7: Production Lead-Time across all Items.

lead time greater than 0.1 (raw processing time), varies significantly

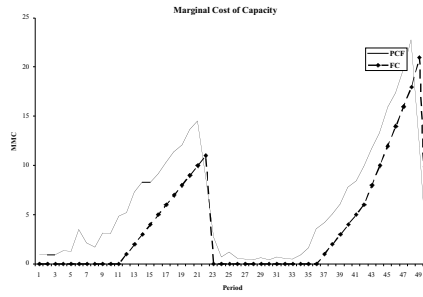


Figure 8: Marginal Cost of Capacity (MCC).

capacity constraint active in ACF, marginal cost positive

marginal cost is zero in FC for all periods where FGI is zero

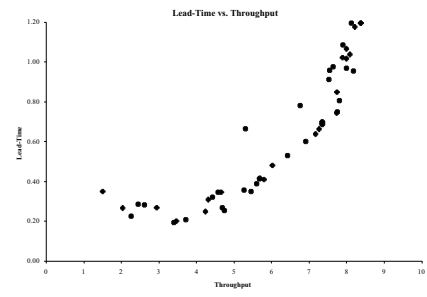


Figure 9: Nonlinear Relationship between throughput and lead time