

Symmetric Traveling Salesman Problem, cuts

Since only interested in inequalities, the problem is a *Hamilton circuit problem*

Graph $G = (V, E)$. For $U \subset V$ we define

- $E(U)$ the set of edges with both ends in U

If $E' \subseteq E$ and $|E' \cap E(U)| \geq |U|$ then the subgraph $G' = (V, E')$ contains at least one subtour.

$$\sum_{e \in E(U)} x_e \leq |U| - 1 \text{ for all } U \subset V,$$

when $2 \leq |U| \leq m - 1$. Where m is the number of nodes.

Symmetric Traveling Salesman Problem

The subtour elimination constraints

$$\sum_{e \in E(U)} x_e \leq |U| - 1 \text{ for all } U \subset V,$$

when $2 \leq |U| \leq m - 1$ define *facets*.

Proof in Nemhauser and Wolsey, p. 274-275.

Traveling Salesman Problem

$$\begin{aligned} & \text{minimize} && \sum_{e \in E} c_e x_e \\ & \text{subject to} && \sum_{e \in \delta(v)} x_e = 2 \text{ for all } v \in V \\ & && \sum_{e \in E(U)} x_e \leq |U| - 1 \text{ for all } U \subset V, \\ & && 2 \leq |U| \leq m - 1 \\ & && x_e \in \{0, 1\} \quad e \in E \end{aligned}$$

Symmetric Traveling Salesman Problem

Additional cuts.

- H subset of nodes $3 \leq |H| \leq |V| - 1$.
- $F \subset E$ odd set of disjointed edges, one end in H .

Chvatal-cut: For all $v \in H$ we have

$$\sum_{e \in \delta(v)} x_e = 2$$

For all $e \in \delta(H) \setminus F$

$$-x_e \leq 0$$

For all $e \in F$

$$x_e \leq 1$$

Add constraints with multipliers $\frac{1}{2}$

$$\frac{1}{2} \sum_{v \in H} \sum_{e \in \delta(v)} x_e - \frac{1}{2} \sum_{e \in \delta(H) \setminus F} x_e + \frac{1}{2} \sum_{e \in F} x_e \leq |H| + \frac{|F|}{2}$$

since LHS integral, round down RHS.

Simplifying LHS:

If all edges incident to a $v \in H$ are added

- Interior edge: counted twice
- Exterior edge $\in F$: counted once
- Exterior edge $\notin F$: counted once

Thus

$$\sum_{v \in H} \sum_{e \in \delta(v)} x_e = 2 \sum_{e \in E(H)} x_e + \sum_{e \in \delta(H) \setminus F} x_e + \sum_{e \in F} x_e$$

Insert into equation at previous page, getting:

$$\sum_{e \in E(H)} x_e + \sum_{e \in F} x_e \leq |H| + \lfloor |F|/2 \rfloor$$

which is a valid inequality. (*2-matching inequality*)