A football pool problem: beat the computer!

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A football match can have three outcomes, "home win", "draw", and "away win", represented on a pool's coupon by "1", "×", and "2", respectively.

To win on the pools a gambler is normally interested in maximizing the number of correct guesses. The inverse problem, however, was the subject of a brain teaser published in a Danish journal in 2003: "For a tournament with 12 matches, what is the smallest number ω_{12} of coupons to be filled in such that at least one has 12 incorrect guesses?". A bottle of Scotch was offered for the best answer presented in one week. Having received no answers and using an invalid argument, the originator of the problem announced his own solution, $\omega_{12} = 512$, and cashed the award.

For a 0-1 matrix $\Delta = \{\delta_{ij}\}$, column j is said to *cover* row i if $\delta_{ij}=1$. A *cover* is a subset of columns covering all rows. *Unweighted SET COVER (USC)* is the problem of finding a cover containing as few columns as possible.

Not much reflection is needed to realize that the solution of a highly structured instance Δ of USC will answer the question posed. The instance Δ or rather, the *family* of instances Δ_n, is a series of square matrices of size 3^n, n=1,2, ... , where n is the number of matches.

Let ω_n be the number of columns in an optimal solution to Δ_n. Some preliminary investigations have enabled us to determine ω_n, n=1,2,3,4 and to show that ω_5 must be either 12 or 13.

12 or 13? Unfortunately LP-based bounds take us nowhere in this case since the optimum is flat as a pancake. It offers some consolation though that experiments with *CPLEX* were not too encouraging either. For n=5, Δ_5 is a matrix of size 243×243. Nothing was returned after 24 hours CPU time. Eventually, upon an investment of 72 hours CPU time, CPLEX managed to come up with ω_5=12.

The original problem asks for ω_{12}, that is, an optimal solution to a square matrix of size 531,441. Since further experiments with CPLEX are unlikely to work we have so far via a "paper-pencil-puzzle approach" devised lower and upper bounds on ω_n and shown that the lower bounds are tight for all n<8. It is furthermore *conjectured* that this property applies for *all* values of n. If true, ω_{12} = 210.