A football pool problem: beat the computer!

Jakob Krarup

DIKU (Dept. of Computer Science, University of Copenhagen)
Universitetsparken 1, DK-2100 Copenhagen, Denmark

John Villadsen

BioCentrum-DTU
DK-2800 Kgs. Lyngby, Denmark

A football match can have three outcomes, "home win", "draw", and "away win", represented on a pool's coupon by "1", "x", and "2", respectively.

To win on the pools a gambler is normally interested in maximizing the number of correct guesses. The inverse problem, however, was the subject of a brain teaser published in a Danish journal in 2003: "For a tournament with 12 matches, what is the smallest number \( \omega_{12} \) of coupons to be filled in such that at least one has 12 incorrect guesses?". A bottle of Scotch was offered for the best answer presented in one week. Having received no answers and using an invalid argument, the originator of the problem announced his own solution, \( \omega_{12} = 512 \), and cashed the award.

For a 0-1 matrix \( \Delta = (\delta_{ij}) \), column \( j \) is said to cover row \( i \) if \( \delta_{ij} = 1 \). A cover is a subset of columns covering all rows. Unweighted SET COVER (USC) is the problem of finding a cover containing as few columns as possible.

Not much reflection is needed to realize that the solution of a highly structured instance \( \Delta \) of USC will answer the question posed. The instance \( \Delta \) or rather, the family of instances \( \Delta_n \), is a series of square matrices of size \( 3^n \), \( n = 1, 2, \ldots \), where \( n \) is the number of matches.

Let \( \omega_n \) be the number of columns in an optimal solution to \( \Delta_n \). Some preliminary investigations have enabled us to determine \( \omega_n, n = 1, 2, 3, 4 \) and to show that \( \omega_5 \) must be either 12 or 13.

12 or 13? Unfortunately LP-based bounds take us nowhere in this case since the optimum is flat as a pancake. It offers some consolation though that experiments with CPLEX were not too encouraging either. For \( n = 5 \), \( \Delta_5 \) is a matrix of size 243 \( \times 243 \). Nothing was returned after 24 hours CPU time. Eventually, upon an investment of 72 hours CPU time, CPLEX managed to come up with \( \omega_5 = 12 \).

The original problem asks for \( \omega_{12} \), that is, an optimal solution to a square matrix of size 531,441. Since further experiments with CPLEX are unlikely to work we have so far via a "paper-pencil-puzzle approach" devised lower and upper bounds on \( \omega_n \) and shown that the lower bounds are tight for all \( n < 8 \). It is furthermore conjectured that this property applies for all values of \( n \). If true, \( \omega_{12} = 210 \).