Flexibility of Steiner Trees in Uniform Orientation Metrics

Martin Zachariasen

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Exercise 1

Prove directly that two edges in a λ-SMT cannot meet at a Steiner point at an angle of $60°$ for $\lambda = 6$. Hint: Prove that the tree can be shortened by inserting a Steiner point between the two edges.

Exercise 2

Give examples of λ-SMTs for $\lambda = 3$ and $\lambda = 4$ which have no flexibility at all, i.e., the interior of their flexibility polygon is empty. Each example should have at least 3 terminals.

Exercise 3

Consider two arbitrary points $p = (p_x, p_y)$ and $q = (q_x, q_y)$. Assume for a given $\lambda \geq 2$, that a shortest path between p and q in the λ-metric consists of (at most) two line segments having orientations $k\omega$ and $(k+1)\omega$, where $\omega = \pi/\lambda$ and $1 \leq k \leq \lambda$.

Give a *linear program* that computes the λ-distance between p and q under these assumptions. Hint: Set up a linear equality for computing the corner point where the two line segments meet; represent the orientations $k\omega$ and $(k+1)\omega$ by unit vectors.

Exercise 4

Argue shortly how the result from Exercise 3 can be used to compute a λ-SMT for a fixed topology, *given* that the orientations of all edges in the tree are known.