Updated Two Commodity Flow Model

Given a symmetric CVRP problem formulated as a graph problem on the graph $G = (V, E)$ as defined in the slides, we construct a new graph $G' = (V', E')$ from G by creating a copy of the depot as node $n + 1$. Connect node $n + 1$ to all nodes $i \in V$ by creating edges with weights as the edges from node 0 to i. As the problem is symmetric, E' only contains edges (i, j) where $i < j$.

A route in G' is represented as a path starting at node 0 and ending at node $n + 1$.

The variables x_{ij} indicates if a vehicle uses edge (i, j), $i < j$.

The variables y_{ij} defines a *commodity flow*. If a vehicle travels from i to j using edge (i, j) then y_{ij} gives the vehicle load while travelling along the arc and y_{ji} gives the empty space on the vehicle while traversing the edge. Notice that the y_{ij} variables are defined for all $i, j \in V'$.

The two commodity flow model is:

(VRP4)

$$\min \sum_{(i,j)\in E'} c_{ij} x_{ij} \tag{1}$$

Subject to:

$$\sum_{j\in V'} (y_{ji} - y_{ij}) = 2d_i \qquad \forall i \in V' \setminus \{0, n+1\} \tag{2}$$

$$\sum_{j\in V'\setminus\{0,n+1\}} y_{0j} = d(V' \setminus \{0, n+1\}) \tag{3}$$

$$\sum_{j\in V'\setminus\{0,n+1\}} y_{j0} = KC - d(V' \setminus \{0, n+1\}) \tag{4}$$

$$\sum_{j\in V'\setminus\{0,n+1\}} y_{n+1,j} = KC \tag{5}$$

$$y_{ij} + y_{ji} = Cx_{ij} \qquad \forall (i, j) \in E' \tag{6}$$

$$\sum_{j\in V', i<j} x_{ij} + \sum_{j\in V', i>j} x_{ji} = 2 \qquad \forall i \in V' \setminus \{0, n+1\} \tag{7}$$

$$y_{ij} \geq 0 \qquad \forall i, j \in V' \tag{8}$$

$$x_{ij} \in \{0, 1\} \qquad \forall (i, j) \in E' \tag{9}$$