Updated Two Commodity Flow Model

Given a symmetric CVRP problem formulated as a graph problem on the graph $G = (V, E)$ as defined in the slides, we construct a new graph $G' = (V', E')$ from $G$ by creating a copy of the depot as node $n + 1$. Connect node $n + 1$ to all nodes $i \in V$ by creating edges with weights as the edges from node 0 to $i$. As the problem is symmetric, $E'$ only contains edges $(i, j)$ where $i < j$.

A route in $G'$ is represented as a path starting at node 0 and ending at node $n + 1$.

The variables $x_{ij}$ indicates if a vehicle uses edge $(i, j)$, $i < j$.

The variables $y_{ij}$ defines a commodity flow. If a vehicle travels from $i$ to $j$ using edge $(i, j)$ then $y_{ij}$ gives the vehicle load while travelling along the arc and $y_{ji}$ gives the empty space on the vehicle while traversing the edge. Notice that the $y_{ij}$ variables are defined for all $i, j \in V'$.

The two commodity flow model is:

(VRP4)

$$
\text{min} \sum_{(i, j) \in E'} c_{ij} x_{ij}
$$

Subject to:

$$
\sum_{j \in V'} (y_{ji} - y_{ij}) = 2d_i \quad \forall i \in V' \setminus \{0, n + 1\}
$$

$$
\sum_{j \in V' \setminus \{0, n + 1\}} y_{0j} = d(V' \setminus \{0, n + 1\})
$$

$$
\sum_{j \in V' \setminus \{0, n + 1\}} y_{j0} = KC - d(V' \setminus \{0, n + 1\})
$$

$$
\sum_{j \in V' \setminus \{0, n + 1\}} y_{n+1,j} = KC
$$

$$
\sum_{j \in V' \setminus \{0, n + 1\}} y_{ij} + y_{ji} = C x_{ij} \quad \forall (i, j) \in E'
$$

$$
\sum_{j \in V', i < j} x_{ij} + \sum_{j \in V', i > j} x_{ji} = 2 \quad \forall i \in V' \setminus \{0, n + 1\}
$$

$$
y_{ij} \geq 0 \quad \forall i, j \in V'
$$

$$
x_{ij} \in \{0, 1\} \quad \forall (i, j) \in E'
$$