Projektopgave P1

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Udleveringsdato: 8. oktober 2004
Afleveringsfrist: 22. oktober 2004 kl. 14.00

Generelt

Dette er den første af to projektopgaver, der kan løses i grupper på op til tre studerende og hvis besvarelse skal godkendes for at gruppens medlemmer kan indstille sig til den skriftlige eksamen.

En ikke-håndskrevet besvarelse skal indleveres i 2 eksemplarer til DIKUs studieadministration (DIKU, stueetagen).

Grupper, hvis besvarelse ikke kan godkendes, får mulighed for at genaflevere, dog kun under forudsætning af der efter bedømmersens mening er gjort et reelt forsøg på at løse opgaven ved første aflevering. For ikke-godkendte besvarelser vil der senest den 4. november blive givet besked pr. email til de pågældende grupper om genaflevering, og tidsfristen for denne er 11. november kl. 12.00.

1. Modification of the cook’s problem

The cook’s problem (reference: handouts 3, 17.09.04)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>1,000 units per gramme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-vitamin</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>R-vitamin</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Cost, BEF/g</td>
<td>4</td>
<td>7</td>
<td>24</td>
<td>12</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Requirements per portion: Q-vit. ≥ 4,000 units, R-vit. ≥ 3,000 units.

Minimize \( z = \) total cost per portion

Optimal solution: \( x_1 = x_2 = x_5 = 0, \) \( x_3 = 1/2, \) \( x_4 = 3/2, \) \( z = 30 \)

Modification of the cook’s problem: Replace the original requirements by

"Q-vit ≥ (4,000 + \( \delta_q \times 1,000 \)) units",

and "R-vit ≥ (3,000 + \( \delta_r \times 1,000 \)) units"

1.1 For which values of \( \delta_q, \delta_r \) will the previous optimal solution remain optimal?
2. **LP duality**

Consider the following LP-model called (P):

\[
\begin{array}{rrrrr}
\text{(P)} & x_1 & x_2 & x_3 & x_4 \\
\hline
\text{s.t.} & 100 & 90 & 50 & 70 & = z & \text{to be minimized} \\
& 1 & 1 & 1 & 1 & = 1 \\
& 5 & 0 & 5 & 7 & \geq 6 \\
& 3 & 0 & 3 & 5 & \geq 4 \\
& 1 & 0 & 2 & 4 & \leq 2.8 \\
\end{array}
\]

\[x_j \text{ nonnegative, } j=1,\ldots,4\]

Let \(y_1, y_2, \ldots\) be the variables in the dual (D) of (P).

2.1 **Formulate the dual (D) of (P).**

An optimal solution to (P) is \(x_1 = .2; x_2 = 0; x_3 = .3; x_4 = .5\). An LP-solver, however, does also provide the so-called **shadow prices** also known as **simplex multipliers** and **dual prices**. It appears that the shadow prices associated with

\[
\text{constraint } (\text{●}) \quad \text{is } \ 0 \\
\text{and} \quad \text{constraint } (\text{●●}) \quad \text{is } \ 50
\]

2.2 **Solve (D) to optimality and verify that the optimal solutions to (P) and (D) satisfy the Complementary Slackness Conditions.**

3. **Θ-AIR**

Europe is flooded with airline companies. Disregarding companies tailored for highly specialized niche operations, however, it is estimated that Europe in the future will have room for but a handful of major companies, established by merging a number of the existing ones, possibly supported by overseas partners.

Under the code name **Theta Air (Θ-AIR)** where the Greek letter Θ is meant to symbolize the prospective worldwide coverage, various deliberations have begun, though so secretly that no single detail as yet has been revealed to the public.

It is inevitable that a successful outcome of such concerted efforts must be heavily dependent upon decision support provided by the formulation and solution of a wide variety of planning problems. Experts in model building and optimization are hereby invited to join the team.

3.1: **The Θ-AIR fleet**

Buying new aircrafts is essentially a **multicriteria** decision problem since many factors have to be taken into account. Among the determinants are: seating capacity, maximum payload, fuel consumption, cruising speed, range, financing, maintenance, training of personnel, noise pollution and other environmental issues. To simplify matters, however, we shall in the sequel concentrate on costs, payload, and seating capacity only.
For a limited part of the prospective Θ-AIR network, three types A1, A2, A3 of aircrafts seem particularly attractive. The relevant data are:

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (millions DKK)</td>
<td>208</td>
<td>162</td>
<td>192</td>
</tr>
<tr>
<td>Maximum payload (tons)</td>
<td>80</td>
<td>90</td>
<td>120</td>
</tr>
<tr>
<td>Number of seats</td>
<td>260</td>
<td>180</td>
<td>160</td>
</tr>
</tbody>
</table>

Θ-AIR will need a fleet which can carry a total payload of at least 3000 tons and provide seats for at least 4000 passengers. Let \( x_1, x_2, x_3 \) be the number of aircrafts to be purchased of each type.

We disregard the fact that aircrafts by nature are "indivisible entities" and hence, that our decision variables ought to be nonnegative integers. Under this assumption, \( x_1, x_2, x_3 \) can take any nonnegative values.

The objective is clearly to find values of \( x_1, x_2, x_3 \) such that both requirements are met at least total cost \( z_{\text{OLP-P}} \). \( \text{OLP-P} \) is the corresponding model.

3.1 What are the values of \( x_1, x_2, x_3 \), and \( z_{\text{OLP-P}} \) in an optimal solution to \( \text{OLP-P} \)?

3.2-3.3: Θ-CHARTER

Θ-CHARTER is a sister company to Θ-AIR offering air transportation to a number of travel agencies specializing in charter tourism. Based on a careful screening of the market, Θ-CHARTER wishes to operate with a fleet of aircrafts corresponding to a total capacity of at least 3000 tons of cargo and at least 4000 passenger seats. To this end, Θ-CHARTER can either lease aircrafts from Θ-AIR in which case Θ-AIR will charge certain amounts

\[ y_p \text{ (DKK 100,000 per ton of cargo)}, \quad \text{and} \quad y_s \text{ (DKK 100,000 per passenger seat)} \]

Alternatively, Θ-CHARTER can buy its own aircrafts of types A1, A2, and A3 under the same conditions as are applying for Θ-AIR, cf. the table above.

\( y_p \) and \( y_s \) are now to be negotiated between the two parties involved. Since Θ-AIR and Θ-CHARTER can be viewed as independent "cost units" although they belong to the same consortium, it is agreed that Θ-AIR should seek its profit maximized whereas the price charged to Θ-CHARTER must be competitive in the sense that it must not exceed the price at which Θ-CHARTER can purchase its own fleet of aircrafts.

3.2 Let \( \text{OLP-D} \) be the model determining the optimal values of \( y_p \) and \( y_s \). Obviously, \( \text{OLP-D} \) must include three constraints which we, with reference to the three aircrafts, designate by (C-A1), (C-A2), and (C-A3), respectively. Solve \( \text{OLP-D} \) to optimality. Let \( w_{\text{OLP-D}} \) be the resulting maximum value of the objective function.

3.3 Replace "192" in the (C-A3) constraint of \( \text{OLP-D} \) by "193" and resolve the problem. Let \( w_{\text{OLP-D}(193)} \) be the new value of the objective function. What is the value of the difference

\[ w_{\text{OLP-D}} - w_{\text{OLP-D}(193)} \]
3.4-3.5: Aircraft A02

Assume that the A2 aircraft is withdrawn and no longer can be considered by either Θ-AIR or Θ-CHARTER. Instead, an older model A02 is being offered:

Cost (millions DKK): 242, maximum payload: 110 tons, number of seats: 220.

Reinsert "192" in the (C-A3)-constraint of ΘLP-D. Replace the (C-A2)-constraint in ΘLP-D by the new one called (C-A02).

3.4 Is (C-A02) a redundant constraint, that is, does its removal not affect the feasible region?

3.5 With aircraft A2 replaced by aircraft A02, what is an optimal composition of the Θ-AIR fleet?

3.6-3.7: The route network

Based upon preliminary deliberations of various kinds, Θ-AIR has eventually decided to open terminals $T_1, T_2, \ldots, T_n$ in towns 1,...,n and to have nonstop flights between terminals $T_i$ and $T_j$ if $s-r > 1$, where $(r,s)$ is any pair of distinct numbers satisfying $1 \leq r < s \leq n$.

Each of these nonstop flights between a pair of terminals (that is, from $T_i$ to $T_j$ and from $T_j$ to $T_i$) is represented by an edge in the route network $R_n$ having the n terminals as its vertices.

3.6 As a modest first step, take $n=5$ and establish terminals $T_1, \ldots, T_5$. What does the corresponding Θ-AIR route network $R_5$ then look like?

3.7 A map of $R_n$, the Θ-AIR route network for $n=6$ is shown in fig. 1:

![Fig. 1: The Θ-AIR route network $R_6$](image)

Unfortunately, instead of being identified by their numbers, terminals $T_1, T_2, T_3, T_4$ in some order have been designated by the letters A,B,C,D as shown. In which order do these four terminals appear in fig. 1?
4. Linear Assignment

Given:

<table>
<thead>
<tr>
<th>82</th>
<th>69</th>
<th>33</th>
<th>80</th>
<th>72</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>13</td>
<td>19</td>
<td>41</td>
<td>28</td>
<td>43</td>
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<td>88</td>
<td>35</td>
<td>38</td>
<td>60</td>
<td>74</td>
<td>63</td>
</tr>
</tbody>
</table>

4.1 From the matrix above, select 6 elements, one from each row and each column such that the sum of the elements selected is minimized.