VL SI-design

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Exercise 1: NP-hardness (30 min, mandatory)

The BIN-PACKING problem asks to cut $n$ pieces, each having width $w_i$, from the minimum number of stocks (bins) each having width $W$. The model may be formulated as

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{n} y_i \\
\text{subject to} & \quad \sum_{j=1}^{n} w_j x_{ij} \leq W y_i \quad i = 1, \ldots, n \\
& \quad \sum_{i=1}^{n} x_{ij} \geq 1 \quad j = 1, \ldots, n \\
& \quad y_j, x_{ij} \in \{0, 1\} \quad i, j = 1, \ldots, n
\end{align*}
\]  

(1)

where $x_{ij} = 1$ iff piece $j$ is cut from stock (bin) $i$, and $y_i = 1$ iff stock (bin) $i$ is used.

The BIN-PACKING-DECISION problem asks whether the $n$ pieces can be cut from $k$ stocks (bins) of width $W$. It has been shown in the literature that BIN-PACKING-DECISION is NP-complete.

The 2D-RECTANGULAR PACKING DECISION problem asks to pack $n$ rectangles of dimensions $w_i \times h_i$ into a rectangle of dimensions $W \times H$.

a) Prove that 2D-RECTANGULAR PACKING DECISION is NP-complete by reduction from BIN-PACKING-DECISION.

Exercise 2: 2D rectangular slicing tree (30 min)

Any 2D rectangular slicing floorplan can be described as a slicing tree. The slicing floorplan can also be described in reverse polish notation:
- \( xyH \) means that \( x \) is separated from \( y \) by a horizontal cut
- \( xyV \) means that \( x \) is separated from \( y \) by a vertical cut

E.g. the slicing floorplan in exercise 3 can be written as

\[ beVdHeV\alpha H \]

a) What is the size of the solution space for 2D rectangular slicing packings.

**Exercise 3: Sequence pairs and corner block lists (30 min, mandatory)**

\[
\begin{array}{c|cc}
| & d & \\
\hline
| c & b & e \\
| & a & \\
\end{array}
\]

a) Write the above packing as a sequence pair.

b) Write the above packing as an O-tree.

**Exercise 4: \( B^* \) tree (30 min)**

Consider the packing from exercise 3.

a) Write the packing as a \( B^* \)-tree.