A Logical Labyrinth

The puzzle is taken from *The Lady or the Tiger* (by Smullyan 1982, formulated as IP-model by Martin J. Chlond, Cath M.Toase). The relevant chapter, *Ladies or Tigers*, contains 12 puzzles of increasing difficulty. In each puzzle a prisoner is faced with a decision where he must open one of several doors. In the first few examples each room contains either a lady or a tiger and in the more difficult examples rooms may also be empty. The following puzzle is the most difficult.

If the prisoner opens a door to find a lady he will marry her and if he opens a door to find a tiger he will be eaten alive. We assume that the prisoner would prefer to be married than eaten alive. It is also assumed that the lady is in some way special to the prisoner and he would prefer to find and marry her rather than an open a door into and empty room.

Each of the doors has a sign bearing a statement that may be either true or false. The puzzle involves nine rooms. The statements on the nine doors are:

- Door 1: The lady is in an odd-numbered room
- Door 2: This room is empty
- Door 3: Either sign 5 is right or sign 7 is wrong
- Door 4: Sign 1 is wrong
- Door 5: Either sign 2 or sign 4 is right
- Door 6: Sign 3 is wrong
- Door 7: The lady is not in room 1
- Door 8: This room contains a tiger and room 9 is empty
- Door 9: This room contains a tiger and sign 6 is wrong

In addition, the prisoner is informed that only one room contains a lady; each of the others either contains a tiger or is empty. The sign on the door of the room containing the lady is true, the signs on all the doors containing tigers are false, and the signs on the doors of empty rooms can be either true or false.

The puzzle as stated does not have a unique solution until the prisoner is told whether or not room eight is empty and this knowledge enables him to find a unique solution.

Define subscripts $i = 1,\ldots,9$ and $j = 1,\ldots,3$ where (1 lady, 2 tiger, 3 empty) and as above variables are

$$x_{i,j} = \begin{cases} 1 & \text{if door } i \text{ hides prize } j \\ 0 & \text{otherwise} \end{cases}$$

$$t_i = \begin{cases} 1 & \text{if statement on door } i \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

1 Formulate the model as an IP model.

2 Use CPLEX to solve the model.

3 What was the additional information the prisoner needed to know to find a unique solution?